MECHANICAL DRAWING

MATHEMATICS

I. C. S. REFERENCE

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A TEXTBOOK

ON

MECHANICAL DRAWING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ARITHMETIC
MENSURATION
ANSWERS TO QUESTIONS

WITH PRACTICAL QUESTIONS AND EXAMPLES

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PREFACE.

In conseqence of a large and increasing demand on the part of our students for a Drawing Volume that would be uniform in shape with the other volumes, we have had the papers on Drawing reset and have had the Drawing Plates inserted on guards in such a manner that they may be opened to the full width. By the present arrangement the value of the Drawing Volume as a work of reference is greatly increased, and we have no doubt but that our students will greatly appreciate the change in form.

As at present constituted, the papers on Drawing are very thorough and complete, and the student receives all the knowledge of this subject that is necessary for him to know in order to make a drawing of any construction that is likely to be given him in practice. The paper on Mechanical Drawing was recently revised and a special section on the subject of Reading Drawings is included in this paper. The student should experience but little difficulty in reading nearly every drawing that may be placed before him.

The set consists of two volumes, one of which contains the Instruction Papers and Examination Questions entitled Arithmetic and Mensuration. The answers to the Examination Questions are also included in this volume. The other volume contains the Instruction Papers entitled Geometrical Drawing, Mechanical Drawing, Practical Projection, and Development of Surfaces. The Drawing Plates referred to in the text of these papers have been inserted throughout this volume in their proper places.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

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ARITHMETIC.

(SECTION 1.)

DEFINITIONS.

1. Arithmetic is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* boy, *one* horse, *one* dozen.

3. A **number** is a unit or a collection of units, as *one*, *three* apples, *five* boys.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *horses*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. Like numbers are numbers which express units of the *same kind*, as 6 *days* and 10 *days*, 2 *feet* and 5 *feet*.

8. Unlike numbers are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *dollars* and five *feet*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. Notation is the art of expressing numbers by figures or letters.

11. Numeration is the art of reading the numbers which have been expressed by figures or letters.

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12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero.*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and, when standing alone, has no value.

The other nine figures are called **digits,** and each one has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each *figure* must express a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local** or **relative** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local** or **relative** value is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus. 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus. 66

The original figure is still *six units*, but the second one is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions, six hundred sixty-six thousands, six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local* or *relative* value. Each figure is *ten times* greater in value than the one immediately on its *right.*

19. The **cipher** (0) has no value itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred*, and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens.* To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens.* *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds and tens;* thus, 450. *Four thousand and fifty* would be expressed 4050, the first cipher indicating that there are no hundreds and the second that there are no units.

NOTE.—When speaking of the figures of a number by referring to them as first figure, second figure, etc., always begin to count at the *left.* Thus, in the number 41,625, 4 is the first figure, 6 the third figure, 5 the fifth or last figure, etc.

20. In *reading* figures, it is usual to point off the number into groups of three figures each, beginning with the right-hand or **units** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2 ,	1	9	8 ,	7	6	5 ,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units*, *tens*, *hundreds;* the next group of three figures is *thousands*, therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma; we next read *millions*, *tens of millions*, *hundreds of millions*, and insert another comma; we then read *billions*, *tens of billions*, *hundreds of billions.*

The entire line of figures would be read: *Four hundred thirty-two billions, one hundred ninety-eight millions, seven hundred sixty-five thousands, four hundred thirty-two.* When we thus *read* a line of figures it is called **numeration,** and if the **numeration** be changed back to *figures*, it is called **notation.**

For instance, the writing of the figures,

72,584,623,

would be the **notation,** and the **numeration** would be *seventy-two millions, five hundred eighty-four thousands, six hundred twenty-three.*

21. Note.—It is customary to leave the *s* off the words millions, thousands, etc., in cases like the above, both in speaking and writing; hence, the above would usually be expressed, seventy-two million, five hundred eighty-four thousand, six hundred twenty-three.

22. The four fundamental processes of Arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes, because all operations in Arithmetic are based upon them.

ADDITION.

23. **Addition** is the *process* of *finding* the *sum* of *two* or *more* numbers. The sign of addition is $+$. It is read *plus*, and means *more.* Thus, $5+6$ is read 5 *plus* 6, and means that 5 and 6 are to be added.

24. The sign of equality is $=$. It is read *equals* or *is equal to.* Thus, $5+6=11$ may be read 5 plus 6 *equals* 11.

25. *Like numbers* can be added, but *unlike numbers* cannot. Thus, 6 dollars *can* be added to 7 dollars, and the *sum* will be 13 dollars, but 6 dollars *cannot* be added to 7 *feet.*

26. The following table gives the sum of any two numbers from 1 to 12:

TABLE 1.

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4	4 and 1 are 5
1 and 2 are 3	2 and 2 are 4	3 and 2 are 5	4 and 2 are 6
1 and 3 are 4	2 and 3 are 5	3 and 3 are 6	4 and 3 are 7
1 and 4 are 5	2 and 4 are 6	3 and 4 are 7	4 and 4 are 8
1 and 5 are 6	2 and 5 are 7	3 and 5 are 8	4 and 5 are 9
1 and 6 are 7	2 and 6 are 8	3 and 6 are 9	4 and 6 are 10
1 and 7 are 8	2 and 7 are 9	3 and 7 are 10	4 and 7 are 11
1 and 8 are 9	2 and 8 are 10	3 and 8 are 11	4 and 8 are 12
1 and 9 are 10	2 and 9 are 11	3 and 9 are 12	4 and 9 are 13
1 and 10 are 11	2 and 10 are 12	3 and 10 are 13	4 and 10 are 14
1 and 11 are 12	2 and 11 are 13	3 and 11 are 14	4 and 11 are 15
1 and 12 are 13	2 and 12 are 14	3 and 12 are 15	4 and 12 are 16
5 and 1 are 6	6 and 1 are 7	7 and 1 are 8	8 and 1 are 9
5 and 2 are 7	6 and 2 are 8	7 and 2 are 9	8 and 2 are 10
5 and 3 are 8	6 and 3 are 9	7 and 3 are 10	8 and 3 are 11
5 and 4 are 9	6 and 4 are 10	7 and 4 are 11	8 and 4 are 12
5 and 5 are 10	6 and 5 are 11	7 and 5 are 12	8 and 5 are 13
5 and 6 are 11	6 and 6 are 12	7 and 6 are 13	8 and 6 are 14
5 and 7 are 12	6 and 7 are 13	7 and 7 are 14	8 and 7 are 15
5 and 8 are 13	6 and 8 are 14	7 and 8 are 15	8 and 8 are 16
5 and 9 are 14	6 and 9 are 15	7 and 9 are 16	8 and 9 are 17
5 and 10 are 15	6 and 10 are 16	7 and 10 are 17	8 and 10 are 18
5 and 11 are 16	6 and 11 are 17	7 and 11 are 18	8 and 11 are 19
5 and 12 are 17	6 and 12 are 18	7 and 12 are 19	8 and 12 are 20
9 and 1 are 10	10 and 1 are 11	11 and 1 are 12	12 and 1 are 13
9 and 2 are 11	10 and 2 are 12	11 and 2 are 13	12 and 2 are 14
9 and 3 are 12	10 and 3 are 13	11 and 3 are 14	12 and 3 are 15
9 and 4 are 13	10 and 4 are 14	11 and 4 are 15	12 and 4 are 16
9 and 5 are 14	10 and 5 are 15	11 and 5 are 16	12 and 5 are 17
9 and 6 are 15	10 and 6 are 16	11 and 6 are 17	12 and 6 are 18
9 and 7 are 16	10 and 7 are 17	11 and 7 are 18	12 and 7 are 19
9 and 8 are 17	10 and 8 are 18	11 and 8 are 19	12 and 8 are 20
9 and 9 are 18	10 and 9 are 19	11 and 9 are 20	12 and 9 are 21
9 and 10 are 19	10 and 10 are 20	11 and 10 are 21	12 and 10 are 22
9 and 11 are 20	10 and 11 are 21	11 and 11 are 22	12 and 11 are 23
9 and 12 are 21	10 and 12 are 22	11 and 12 are 23	12 and 12 are 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 are 17.

27. For *addition*, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure* of *one number* is placed *directly under the right-hand figure*

of the *number above it*, thus bringing the unit figures of all the numbers to be added in the same vertical line. Proceed as in the following examples:

28. EXAMPLE.—What is the sum of 131, 222, 21, 2, and 413?

SOLUTION.—

131
222
21
2
413
sum 789 Ans.

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the right-hand or *units* column, and add, mentally repeating the different sums. Thus, three and two are five and one are six and two are eight and one are nine, the sum of the numbers in *units* column. Place the 9 directly beneath as the first or *units* figure in the sum.

The sum of the numbers in the next or *tens* column equals 8 *tens*, which is the second or *tens* figure in the sum.

The sum of the numbers in the next or *hundreds* column equals 7 *hundreds*, which is the third or *hundreds* figure in the sum.

The sum or answer is 789.

29. EXAMPLE.—What is the sum of 425, 36, 9,215, 4, and 907?

SOLUTION.—

425
36
9215
4
907
27
60
1500
9000
sum 10587 Ans.

EXPLANATION.—The sum of the numbers in the first or units column is seven and four are eleven and five are sixteen and six are twenty-two and five are twenty-seven, or 27 units; i. e., two tens and seven units. Write 27 as shown.

The sum of the numbers in the second or tens column is six tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third or hundreds column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth or thousands column, nine, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens, when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column above, the sum of 16 and 6 was required. We know from the table that $6 + 6 = 12$; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than thirty, since $20 + 10 = 30$; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number and, if the result is greater than 9, increase the second figure of the larger number by 1. Thus, $44 + 7 = ?$ $4 + 7 = 11$; hence, $44 + 7 = 51$.

30. The addition may also be performed as follows:

425
36
9215
4
907
sum 10587 Ans.

EXPLANATION.—The sum of the numbers in *units* column = 27 *units*, or 2 *tens* and 7 *units*. Write the 7 *units* as the first or right-hand figure in the sum. Reserve the two *tens* and add them to the figures in *tens* column. The sum of the figures in the *tens* column, plus the 2 *tens* reserved and carried from the *units* column = 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 *hundreds*, or 1 *thousand* and 5 *hundreds*. Write down the 5 as the third or *hundreds* figure in the sum and carry the 1 to the next column. $1 + 9 = 10$, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below:

SOLUTION.—

890
82
90
393
281
80
770
83
492
80
383
84
191
sum 3899 Ans.

EXPLANATION.—The sum of the digits in the first column equals 19 *units*, or 1 *ten* and 9 *units*. Write down the 9 and carry 1 to the next column. The sum of the digits in the second column + 1 = 109 *tens*, or 10 *hundreds* and 9 *tens*. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 reserved = 38.

The entire sum is 3,899.

32. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column, and add the remaining figure or figures to the next column.*

33. Proof.—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of

(*a*) 104 + 203 + 613 + 214.
(*b*) 1,875 + 3,143 + 5,826 + 10,832.
(*c*) 4,865 + 2,145 + 8,173 + 40,084.
(*d*) 14,204 + 8,173 + 1,065 + 10,042.

Ans. (*a*) 1,134. (*b*) 21,676. (*c*) 55,267. (*d*) 33,484.

(*e*) 10,832 + 4,145 + 3,133 + 5,872.
(*f*) 214 + 1,231 + 141 + 5,000.
(*g*) 123 + 104 + 425 + 126 + 327.
(*h*) 6,354 + 2,145 + 2,042 + 1,111 + 3,333.

Ans. (*e*) 23,982. (*f*) 6,586. (*g*) 1,105. (*h*) 14,985.

SUBTRACTION.

35. In Arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the *subtrahend* from the *minuend* is called the **difference,** or **remainder.**

36. The sign of subtraction is —. It is read **minus,** and means *less*. Thus, 12 — 7 is read 12 *minus* 7, and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend	7568
subtrahend	3425
remainder	4143 Ans.

EXPLANATION.—Begin at the right-hand or *units* column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the *minuend* than in the *subtrahend*, and when some figures in the *minuend* are *less* than the figures directly under them in the *subtrahend*, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—

minuend	8453
subtrahend	844
remainder	7609 Ans.

EXPLANATION.—Begin to subtract at the right-hand or *units* column. We cannot take 4 from 3, and must, therefore, borrow 1 from 5 in *tens* column and annex it to the 3 in

units column. The 1 *ten* = 10 *units*, which added to the 3 in *units* column = 13 *units*. 4 from 13 = 9, the first or *units* figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second or *tens* figure. We cannot take 8 from 4, and must, therefore, borrow 1 from 8 in *thousands* column. Since 1 *thousand* = 10 *hundreds*, 10 *hundreds* + 4 *hundreds* = 14 *hundreds*, and 8 from 14 = 6, the third or *hundreds* figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder or answer.

The operation of borrowing is placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend 10000
subtrahend 8763
remainder 1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and taken it to the *units* column, only 9 remains, from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. Rule.—*Place the subtrahend or smaller number under the minuend or larger number, in the same manner as for addition, and proceed as in Arts.* ***37, 38,*** *and* ***39.***

41. Proof.—*To prove an example in subtraction, add the remainder to the subtrahend. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend 8763
remainder 1237
minuend 10000

EXAMPLES FOR PRACTICE.

42. From

(*a*) 94,278 take 62,574.
(*b*) 53,714 take 25,824.
(*c*) 71,832 take 58,109.
(*d*) 20,804 take 10,408.
(*e*) 310,465 take 102,141.
(*f*) (81,043 + 1,041) take 14,831.
(*g*) (20,482 + 18,216) take 21,214.
(*h*) (2,040 + 1,213 + 542) take 3,791.

Ans. (*a*) 31,704. (*b*) 27,890. (*c*) 13,723. (*d*) 10,396. (*e*) 208,324. (*f*) 67,253. (*g*) 17,484. (*h*) 4.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. Multiplication is the process of multiplying one number by another.

The *number* thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The *number* which shows how many times the *multiplicand* is to be taken, or the *number* by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. The sign of multiplication is ×. It is read *times* or *multiplied by*. Thus, 9 × 6 is read 9 *times* 6, or 9 *multiplied by* 6.

46. It matters not in what order the numbers to be multiplied together are placed. Thus, 6 × 9 is the same as 9 × 6.

47. In the following table, the product of any two numbers (neither of which exceeds twelve) may be found:

TABLE 2.

1 times 1 is 1	2 times 1 are 2	3 times 1 are 3
1 times 2 are 2	2 times 2 are 4	3 times 2 are 6
1 times 3 are 3	2 times 3 are 6	3 times 3 are 9
1 times 4 are 4	2 times 4 are 8	3 times 4 are 12
1 times 5 are 5	2 times 5 are 10	3 times 5 are 15
1 times 6 are 6	2 times 6 are 12	3 times 6 are 18
1 times 7 are 7	2 times 7 are 14	3 times 7 are 21
1 times 8 are 8	2 times 8 are 16	3 times 8 are 24
1 times 9 are 9	2 times 9 are 18	3 times 9 are 27
1 times 10 are 10	2 times 10 are 20	3 times 10 are 30
1 times 11 are 11	2 times 11 are 22	3 times 11 are 33
1 times 12 are 12	2 times 12 are 24	3 times 12 are 36
4 times 1 are 4	5 times 1 are 5	6 times 1 are 6
4 times 2 are 8	5 times 2 are 10	6 times 2 are 12
4 times 3 are 12	5 times 3 are 15	6 times 3 are 18
4 times 4 are 16	5 times 4 are 20	6 times 4 are 24
4 times 5 are 20	5 times 5 are 25	6 times 5 are 30
4 times 6 are 24	5 times 6 are 30	6 times 6 are 36
4 times 7 are 28	5 times 7 are 35	6 times 7 are 42
4 times 8 are 32	5 times 8 are 40	6 times 8 are 48
4 times 9 are 36	5 times 9 are 45	6 times 9 are 54
4 times 10 are 40	5 times 10 are 50	6 times 10 are 60
4 times 11 are 44	5 times 11 are 55	6 times 11 are 66
4 times 12 are 48	5 times 12 are 60	6 times 12 are 72
7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
7 times 12 are 84	8 times 12 are 96	9 times 12 are 108
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

This table should be carefully committed to memory.
Since 0 has no value, the product of 0 and any number is 0

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	425	
multiplier	5	
product	2125	Ans.

EXPLANATION.—For convenience, the *multiplier* is generally written *under* the *right-hand figure* of the *multiplicand*. On looking in the multiplication table, we see that 5×5 are 25. *Multiplying* the *first figure* at the *right* of the *multiplicand*, or 5, by the *multiplier* 5, it is seen that 5 times 5 units are 25 units, or 2 tens and 5 units. Write the *5 units* in *units place* in the *product*, and *reserve* the 2 tens to *add* to the *product* of *tens*. Looking in the multiplication table again, we see that 5×2 are 10. *Multiplying* the *second figure* of the *multiplicand* by the *multiplier* 5, we see *that* 5 *times* 2 tens are 10 tens, *plus* the 2 tens *reserved*, are 12 tens, or 1 hundred plus 2 tens. Write the *2 tens* in tens place, and *reserve* the 1 hundred to *add* to the product of *hundreds*. Again, we see by the multiplication table that 5×4 are 20. *Multiplying* the *third* or *last figure* of the *multiplicand* by the *multiplier* 5, we see that 5 times 4 hundreds are 20 hundreds, *plus* the 1 hundred *reserved*, are 21 hundreds, or 2 thousands *plus* 1 hundred, which we write in *thousands* and *hundreds places*, respectively.

Hence, the *product* is 2,125.

This *result* is the same as adding 425 five times. Thus,

425
425
425
425
425
sum 2125 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of

(*a*) 61,483 × 6.
(*b*) 12,375 × 5.
(*c*) 10,426 × 7.
(*d*) 10,835 × 3.

Ans. (*a*) 368,898. (*b*) 61,875. (*c*) 72,982. (*d*) 32,505.

(*e*) 98,376 × 4.
(*f*) 10,873 × 8.
(*g*) 71,543 × 9.
(*h*) 218,734 × 2.

Ans. { (*e*) 393,504. (*f*) 86,984. (*g*) 643,887. (*h*) 437,468.

50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

multiplicand	475
multiplier	234
	1900
	1425
	950
product	111150 Ans.

EXPLANATION.—For convenience, the *multiplier* is generally written *under* the *multiplicand*, placing units under units, tens under tens, etc.

We *cannot* multiply by 234 at one operation; we must, therefore, *multiply* by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial product;* 3 times 475 = 1,425, the *second partial product*, the *right-hand figure* of which is *written directly under* the *figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the *right-hand figure* of which is *written directly under* the *figure multiplied by*, or 2.

The sum of these *three partial products* is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply the entire multiplicand by it; since the result will be zero, place a cipher under the cipher in the multiplier. Thus,

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
×0	×0	× 0	× 0
0 Ans.	0 Ans.	0 Ans.	0 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
62280	120240	3126400
632142 Ans.	1222440 Ans.	31326528 Ans.

In examples (*e*), (*f*), and (*g*), we multiply by 0 as directed above; then multiply by the next figure of the multiplier and place the first figure of the product alongside the 0, as shown.

EXAMPLES FOR PRACTICE.

54. Find the product of

(*a*) 3,842 × 26.
(*b*) 3,716 × 45.
(*c*) 1,817 × 124.
(*d*) 675 × 38.
(*e*) 1,875 × 33.
(*f*) 4,836 × 47.
(*g*) 5,682 × 543.
(*h*) 3,257 × 246.
(*i*) 2,875 × 302.
(*j*) 17,819 × 1,004.
(*k*) 38,674 × 205.
(*l*) 18,304 × 100.
(*m*) 7,834 × 10.
(*n*) 87,543 × 1,000.
(*o*) 48,763 × 100.

Ans. { (*a*) 99,892. (*b*) 167,220. (*c*) 225,308. (*d*) 25,650. (*e*) 61,875. (*f*) 227,292. (*g*) 3,085,326 (*h*) 801,222. (*i*) 868,250. (*j*) 17,890,276. (*k*) 7,928,170. (*l*) 1,830,400. (*m*) 78,340. (*n*) 87,543,000. (*o*) 4,876,300.

DIVISION.

55. Division is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which *shows* how many times the *divisor* is *contained* in the *dividend* is called the **quotient.**

56. The sign of division is $\div$. It is read *divided by.* $54 \div 9$ is read 54 *divided by* 9. Another way to write 54 *divided by* 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$.

In both of these cases 54 is the *dividend* and 9 is the *divisor.*

Division is the *reverse* of **multiplication.**

57. To divide when the divisor consists of but one figure, proceed as in the following example:

EXAMPLE.—What is the quotient of $875 \div 7$?

SOLUTION.—

divisor dividend quotient

7) 8 7 5 (1 2 5 Ans.
7
---
1 7
1 4
---
3 5
3 5
---
remainder 0

EXPLANATION.—7 is contained in 8 *hundreds* 1 *hundred* times. Place the one as the first or left-hand figure of the quotient. Multiply the divisor 7 by the 1 *hundred* of the quotient, and place the product 7 *hundreds* under the 8 *hundreds* in the dividend, and subtract. Beside the remainder 1, bring down the next or *tens* figure of the quotient, in this case 7, making 17 *tens;* 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor 7 by the 2 in the quotient, and subtract the product from 17. Beside the remainder 3, bring down the next or *units* figure of the dividend, in this case 5, making

35 *units.* 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times $7 = 35$, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend

divisor 7) $8^{1}7^{3}5$

quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, once and one remainder; 1 placed before 7 makes 17; 7 is contained in 17, 2 times and 3 over; the 3 placed before 5 makes 35; 7 is contained in 35, 5 times. These partial quotients placed in order as they are found, make the entire quotient 125.

The small figures are placed in the example given to better illustrate the explanation; they are never written when actually performing division in this way.

59. If the *divisor* consists of *2 or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

SOLUTION.—

divisor dividend quotient

63) 2 7 0 2 8 2 6 (4 2 9 0 2 Ans.
2 5 2
---
1 8 2
1 2 6
---
5 6 8
5 6 7
---
1 2 6
1 2 6

0

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial, we must find how many times 63 is contained in 270;

6 is contained in the first two figures of 270, 4 times. Place the 4 as the first or left-hand figure in the quotient. Multiply the divisor 63 by 4, and subtract the product 252 from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor 63 by 2, and subtracting the product 126 from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor 63 by 9 and subtract the product 567 from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.*

V. *If there be a remainder at last, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor, and add the remainder, if there be any, to the product. The result will be the dividend.*

divisor dividend quotient

Thus, 63)4235(67 14/63 **Ans.**

		378
		455
		441
	remainder	14
Proof,	*quotient*	67
	divisor	63
		201
		402
		4221
	remainder	14
	dividend	4235

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*) 126,498 by 58.	Ans.	(*a*) 2,181.
(*b*) 3,207,594 by 767.		(*b*) 4,182.
(*c*) 11,408,202 by 234.		(*c*) 48,753.
(*d*) 2,100,315 by 581.		(*d*) 3,615.
(*e*) 969,936 by 4,008.		(*e*) 242.
(*f*) 7,481,888 by 1,021.		(*f*) 7,328.
(*g*) 1,525,915 by 5,003.		(*g*) 305.
(*h*) 1,646,301 by 381.		(*h*) 4,321.

CANCELATION.

63. Cancelation is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are *those numbers* which, when *multiplied* together, will *equal that number*. Thus, 5 and 3 are factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which cannot be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc., are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to each other, so also are 22, 25, and 21 — all composite numbers.

67. Canceling *equal factors* from *both dividend and divisor* does *not* change the *quotient*.

The *canceling* of a *factor* in *both dividend and divisor* is the *same* as *dividing them both* by the *same number*, which, by the principle of division, does not *change the quotient*.

Write the *numbers* which make the *dividend* above the *line*, and those which make the *divisor* below it.

68. EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24.

SOLUTION.—Placing the dividend over the divisor, and canceling

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the four and write the 1 over it; also, cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}}.$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also, cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{6}{\cancel{24}}}.$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also, cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}}.$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}} = \frac{1 \times 5 \times 10}{1 \times 1} = 50. \quad \text{Ans.}$$

It is usual to omit the 1's when canceling them, instead of writing them as above.

69. Rule.—I. *Cancel the common factors from both the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide

(*a*) $14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.
(*b*) $3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.
(*c*) $8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.
(*d*) $164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.
(*e*) $50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.
(*f*) $48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.
(*g*) $110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.
(*h*) $115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.

Ans. (*a*) 32. (*b*) 250. (*c*) 1. (*d*) 48. (*e*) 5. (*f*) 105. (*g*) 42. (*h*) 5.

ARITHMETIC.

(SECTION 2.)

FRACTIONS.

71. A **fraction** is one or more equal parts of a unit: *One-half*, *one-third*, *two-fifths* are fractions.

72. *Two* numbers are required to express a fraction; one is called the *numerator*, and the other the *denominator.*

73. The numerator is placed above the denominator, with a line between them; as, $\frac{2}{3}$. 3 is the **denominator,** and shows into how many *equal parts* the *unit* or *one* is divided. The **numerator** 2 shows how many of these equal parts are taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half; $\frac{3}{4}$ is read three-fourths; $\frac{3}{8}$ is read three-eighths; $\frac{5}{16}$ is read five-sixteenths; $\frac{29}{47}$ is read twenty-nine-forty-sevenths.

74. In the expression "$\frac{3}{4}$ of an apple," the denominator 4 shows that the apple is to be (or has been) cut into 4 *equal parts*, and the numerator 3 shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the parts, or fourths, of the apple were cut in *two equal pieces*, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, and written $\frac{3}{8}$. It is evident that the *larger the denominator*, the greater is the number of parts into which anything is divided and the less the value of the fraction for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is smaller than $\frac{7}{8}$, because if an object be divided into 9 parts, the parts are smaller than if the same object had been divided into 8 parts; and, since $\frac{1}{9}$ is smaller than $\frac{1}{8}$,

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it is clear that 7 one-ninths is a smaller amount than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

75. The **value** of a fraction is the *numerator* divided by the *denominator;* as, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

76. The line between the *numerator* and *denominator* means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.
$\frac{5}{8}$ is equivalent to $5 \div 8$.

77. The *numerator* and *denominator* of a fraction, when considered together, are called the **terms** of a fraction.

78. The *value* of a fraction whose *numerator* and *denominator* are equal is 1.

$\frac{4}{4}$, or four-fourths, $= 1$.
$\frac{8}{8}$, or eight-eighths, $= 1$.
$\frac{64}{64}$, or sixty-four-sixty-fourths, $= 1$.

79. A **proper fraction** is a fraction whose *numerator* is *less* than its *denominator*. Its *value* is *less* than 1; as, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

80. An **improper fraction** is a fraction whose *numerator equals* or is *greater than* the *denominator*. Its *value* is *one* or *more* than *one;* as, $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

81. A **mixed number** is a *whole* number and a *fraction* united. $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

82. **Reduction of fractions** is the process of changing their form without changing their *value.*

83. A *fraction* is reduced to *higher terms* by *multiplying both terms* of the *fraction* by the *same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The *value* is not changed, since $\frac{3}{4} = \frac{6}{8}$. For, suppose that an object, say an apple, is divided into 8 equal parts. If

these parts be arranged into 4 piles, each containing 2 parts, it is evident that each pile will be composed of the same amount of the entire apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same course of reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

84. To reduce a fraction to an equal fraction having a given denominator:

EXAMPLE.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will, in this case, make the product 96; this number is evidently $96 \div 8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Ans.

85. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the result.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

86. A *fraction* is reduced to *lower terms* by *dividing both terms* by the *same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in Art. **83**; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

87. A *fraction* is reduced to *lowest terms* when its *numerator and denominator* cannot both be *divided* by the *same*

number without a remainder; as, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$. In other words, the numerator and denominator are prime to each other.

EXAMPLES FOR PRACTICE.

88. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.
(*b*) $\frac{24}{132}$ to its lowest terms.
(*c*) $\frac{64}{1000}$ to its lowest terms.
(*d*) $\frac{5}{7}$ to 49ths.
(*e*) $\frac{13}{16}$ to 10,000ths.

Ans. (*a*) $\frac{56}{128}$. (*b*) $\frac{2}{11}$. (*c*) $\frac{8}{125}$. (*d*) $\frac{35}{49}$. (*e*) $\frac{8125}{10000}$.

---

89. To reduce a whole number or mixed number to an improper fraction:

EXAMPLE.—How many *fourths* in 5?

SOLUTION.—Since there are 4 *fourths* in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.—$8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

90. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

EXAMPLES FOR PRACTICE.

91. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{1}{9}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. (*a*) $\frac{33}{8}$. (*b*) $\frac{46}{9}$. (*c*) $\frac{102}{10}$. (*d*) $\frac{151}{4}$. (*e*) $\frac{254}{5}$. (*f*) $\frac{112}{16}$.

---

92. To reduce an improper fraction to a whole or mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.—4 is contained in 21, 5 times and 1 remaining (see Art. **75**); as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

93. Rule.—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

94. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.
(*b*) $\frac{185}{3}$.
(*c*) $\frac{701}{6}$.
(*d*) $\frac{149}{3}$.
(*e*) $\frac{76}{19}$.
(*f*) $\frac{125}{25}$.

Ans. (*a*) $24\frac{1}{6}$. (*b*) $61\frac{2}{3}$. (*c*) $116\frac{5}{6}$. (*d*) $49\frac{2}{3}$. (*e*) 4. (*f*) 5.

95. A **common denominator** of *two* or *more fractions* is a number which will contain (i. e., which may be divided by) all of the *denominators* of the *fractions* without a remainder. The **least common denominator** is the least number that will contain all of the denominators of the fractions without a remainder.

96. To find the least common denominator:

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

$$\begin{array}{r} 2\,)\,4,\ \ 3,\ \ 9,\ 16 \\ \hline 2\,)\,2,\ \ 3,\ \ 9,\ \ 8 \\ \hline 3\,)\,1,\ \ 3,\ \ 9,\ \ 4 \\ \hline 1,\ \ 1,\ \ 3,\ \ 4 \end{array}$$

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide the numbers by some prime number that will divide at least two of them without a remainder (if possible), bringing down to the row below those denominators which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4.

Dividing the third row by 3, the result is 1, 1, 3, 4. Since the remaining numbers are prime to each other, we cease dividing further. The product of all the divisors and of the numbers prime to each other, is $2\times2\times3\times3\times4=144$, which is the required least common denominator.

97. EXAMPLE.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r} 3)\,9,\ 12,\ 18 \\ 3)\,3,\ 4,\ 6 \\ 2)\,1,\ 4,\ 2 \\ \hline 1,\ 2,\ 1 \end{array}$$

$3\times3\times2\times2=36$. Ans.

98. To reduce two or more fractions to fractions having a common denominator:

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

SOLUTION.—The common denominator is any number which will contain 3, 4, and 2. The *least* common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder. $\frac{2}{3}=\frac{8}{12}$, $\frac{3}{4}=\frac{9}{12}$, $\frac{1}{2}=\frac{6}{12}$.

Reducing $\frac{2}{3}$ (see Art. **84**), 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$\frac{2\times4}{3\times4}=\frac{8}{12}$. In the same way we find $\frac{3}{4}=\frac{9}{12}$ and $\frac{1}{2}=\frac{6}{12}$.

99. Rule. — *Divide the common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

100. Reduce to fractions having a common denominator:

(*a*) $\frac{2}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.
(*d*) $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.
(*f*) $\frac{7}{15}$, $\frac{17}{30}$, $\frac{21}{30}$.

Ans.
(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$.

ADDITION OF FRACTIONS.

101. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into two equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now, if we add these parts, the result is $2+4=6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only 1 apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the quarters to eighths, we have $\frac{2}{4}=\frac{4}{8}$, and adding the other 4 eighths, $4+4=8$ eighths. This result is correct, since $\frac{8}{8}=1$. Or, we can, in this case, reduce the eighths to quarters. Thus, $\frac{4}{8}=\frac{2}{4}$; whence, adding $2+2=4$ quarters, a correct result since $\frac{4}{4}=1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

102. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the *denominators*, is 8.

$\frac{1}{2}=\frac{4}{8}$, $\frac{3}{4}=\frac{6}{8}$, $\frac{5}{8}=\frac{5}{8}$.

EXPLANATION.—As the *denominator* tells or indicates the names of the *parts*, the *numerators* only are added in order to obtain the total number of *parts* indicated by the *denominator*. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths =

$\frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{4+6+5}{8}=\frac{15}{8}=1\frac{7}{8}$. Ans.

103. EXAMPLE.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is **16**.

$$\begin{array}{l} 12\frac{3}{4}=12\frac{12}{16} \\ 14\frac{5}{8}=14\frac{10}{16} \\ 7\frac{5}{16}=\ 7\frac{5}{16} \\ \hline \textit{sum}=33+\frac{27}{16}=33+1\frac{11}{16}=34\frac{11}{16}. \end{array}$$

Ans.

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

EXAMPLE.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{r} 17 \\ 13\frac{6}{32} \\ \frac{9}{32} \\ 3\frac{8}{32} \\ \hline \textit{sum}\ 33\frac{23}{32}. \end{array}$$ Ans.

104. Rule I.—*Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number, and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

105. Find the sum of

(*a*) $\frac{4}{6}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $\frac{11}{72}$, $\frac{13}{18}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{23}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{2}{7}$.

Ans. (*a*) $1\frac{7}{12}$. (*b*) $1\frac{3}{15}$. (*c*) $1\frac{3}{16}$. (*d*) $1\frac{51}{72}$. (*e*) $1\frac{29}{66}$. (*f*) $1\frac{5}{9}$. (*g*) $1\frac{7}{23}$. (*h*) 1.

SUBTRACTION OF FRACTIONS.

106. Fractions cannot be *subtracted* without first reducing them to a *common denominator*. This can be shown in the same manner as in the case of addition of fractions.

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\tfrac{3}{8} = \tfrac{6}{16}. \quad \tfrac{13}{16} - \tfrac{6}{16} = \frac{13-6}{16} = \tfrac{7}{16}.$$ Ans.

107. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, since $7 = 6 + 1$, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$, or $6\frac{3}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

108. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$ Ans.

109. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or $8\frac{20}{16}$	
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
difference	$4\frac{13}{16}$	$4\frac{13}{16}$ Ans.

EXPLANATION.—As the *fraction* in the *subtrahend* is greater than the fraction in the *minuend*, it *cannot* be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the *minuend* and *add* it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was *borrowed* from 9, 8 *remains;* 4 from 8 = 4; $4 + \frac{13}{16} = 4\frac{13}{16}$.

110. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

minuend	9 or $8\frac{16}{16}$	
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$
difference	$\frac{13}{16}$	$\frac{13}{16}$ Ans.

EXPLANATION.—As there is no *fraction* in the *minuend* from which to take the *fraction* in the *subtrahend*, *borrow* 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was *borrowed* from 9, only 8 is left. 8 from 8 = 0.

111. Rule I.—*Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

112. Subtract

(*a*) $\frac{10}{24}$ from $\frac{11}{12}$.
(*b*) $\frac{7}{14}$ from $\frac{17}{28}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.
(*d*) $\frac{15}{35}$ from $\frac{45}{70}$.
(*e*) $\frac{15}{16}$ from $\frac{57}{48}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(*g*) $12\frac{1}{8}$ from 27.
(*h*) $5\frac{1}{4}$ from 30.

Ans. (*a*) $\frac{1}{2}$. (*b*) $\frac{3}{28}$. (*c*) $\frac{11}{30}$. (*d*) $\frac{3}{14}$. (*e*) $\frac{1}{4}$. (*f*) $17\frac{1}{4}$. (*g*) $14\frac{7}{8}$. (*h*) $24\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS.

113. In *multiplication* of fractions it is not necessary to *reduce* the *fractions* to fractions having a *common denominator*.

114. Multiplying the *numerator* or *dividing* the *denominator multiplies* the fraction.

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3}{4} \times 4 = \frac{12}{4} = 3$. Ans.

Or $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word "of" in multiplication of fractions means the same as $\times$, or times. Thus,

$$\tfrac{3}{4} \text{ of } 4 = \tfrac{3}{4} \times 4 = 3.$$
$$\tfrac{1}{8} \text{ of } \tfrac{5}{16} = \tfrac{1}{8} \times \tfrac{5}{16} = \tfrac{5}{128}.$$

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$. Ans.

Or $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

115. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

or, by cancelation, $\frac{\cancel{4} \times 7}{\cancel{16}_{4} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

116. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \cancel{32}_{2}} = \frac{3}{8 \times 2} = \frac{3}{16}$. Ans.

117. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\tfrac{39}{4} \times \tfrac{45}{8} = \frac{39 \times 45}{4 \times 8} = \tfrac{1755}{32} = 54\tfrac{27}{32}. \quad \text{Ans.}$$

118. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$15\frac{7}{8}$ or $15\frac{7}{8}$
3 $\quad\quad$ 3
$47\frac{5}{8}$ $\quad\quad$ $45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}$. Ans.

119. Rule.—I. *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number, and add the whole number part to the product of the multiplier and whole number.*

EXAMPLES FOR PRACTICE.

120. Find the product of

(*a*) $7 \times \frac{3}{19}$.
(*b*) $14 \times \frac{5}{16}$.
(*c*) $\frac{21}{32} \times \frac{5}{14}$.
(*d*) $\frac{16}{27} \times 4$.
(*e*) $\frac{19}{18} \times 7$.
(*f*) $17\frac{18}{21} \times 7$.
(*g*) $\frac{105}{224} \times 32$.
(*h*) $\frac{15}{28} \times 14$.

Ans. (*a*) $1\frac{2}{19}$. (*b*) $4\frac{3}{8}$. (*c*) $\frac{15}{64}$. (*d*) $2\frac{10}{27}$. (*e*) $7\frac{7}{18}$. (*f*) 125. (*g*) 15. (*h*) $7\frac{1}{2}$.

DIVISION OF FRACTIONS.

121. In *division* of fractions it is not necessary to *reduce* the *fractions* to fractions having a *common denominator*.

122. Dividing the *numerator* or *multiplying* the *denominator*, *divides* the fraction.

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing* the *numerator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8} \div 3 = \tfrac{2}{8} = \tfrac{1}{4}. \quad \text{Ans.}$$

When *multiplying* the *denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}$. Ans.

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\frac{14}{32} \div 7 = \frac{14 \div 7}{32} = \frac{2}{32} = \frac{1}{16}$. Ans.

123. To **invert** a fraction is to *turn* it *upside down*; that is, make the *numerator* and *denominator change places*. Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

124. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the *denominators* are the same, and one *numerator* is *contained* in the other 3 times. 2. If we now *invert* the *divisor* $\frac{3}{16}$, and ***multiply***, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This brings the *same quotient* as in the first case.

125. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the *denominators* are *not* the same, therefore, we must solve as in the second case.

$$\tfrac{3}{8} \div \tfrac{1}{4} = \tfrac{3}{8} \times \tfrac{4}{1} = \frac{3 \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

126. EXAMPLE.—Divide 5 by $\frac{10}{16}$.

SOLUTION.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\overset{1}{\cancel{5}} \times \overset{8}{\cancel{16}}}{\underset{2}{\cancel{10}}} = 8. \quad \text{Ans.}$$

127. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

128. Rule.—*Invert the divisor, and proceed as in multiplication.*

129. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\frac{3 \times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\frac{8+4}{16}$.

$\frac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it*.

EXAMPLES FOR PRACTICE.

130. Divide

(*a*) 15 by $6\frac{3}{7}$.
(*b*) 30 by $\frac{6}{8}$.
(*c*) 172 by $\frac{4}{5}$.
(*d*) $\frac{14}{18}$ by $1\frac{7}{16}$.
(*e*) $\frac{103}{6}$ by $14\frac{3}{8}$.
(*f*) $\frac{142}{27}$ by $17\frac{1}{9}$.
(*g*) $\frac{14}{18}$ by $\frac{145}{72}$.
(*h*) $\frac{128}{18}$ by $72\frac{1}{3}$.

Ans. (*a*) $2\frac{1}{4}$. (*b*) 40. (*c*) 215. (*d*) $\frac{112}{207}$. (*e*) $1\frac{15}{88}$. (*f*) $\frac{71}{231}$. (*g*) $\frac{56}{145}$. (*h*) $\frac{64}{651}$.

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131. Whenever an expression like one of the three following is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care however to indicate that the denominator when so transferred is a multiplier.

1. $\frac{\frac{3}{4}}{9} = \frac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4} \times 4}{9 \times 4} = \frac{3}{9 \times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\frac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \frac{5}{\frac{3 \times 9}{4}}$; and $\frac{\frac{5}{\frac{3 \times 9}{4}} \times 4}{\frac{3 \times 9}{4} \times 4} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \cancel{310}^{10} \times \cancel{27}^{3} \times \cancel{72}^{6} \times \cancel{2} \times \cancel{6}^{3}}{\cancel{40}^{4} \times \cancel{9} \times \cancel{31} \times \cancel{4}^{2} \times \cancel{12}^{2}} =$$

$\frac{27}{2} = 13\frac{1}{2}$.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

ARITHMETIC.

(SECTION 3.)

DECIMALS.

REMARK.—A knowledge of decimals is of the utmost importance to all who are required to make calculations of any kind. The subject is easy to learn, and for this reason the student is somewhat inclined to study it too hastily, the result being that he afterwards has trouble that might have been entirely avoided had he given the text the proper attention in the beginning. Decimals are much easier to use than common fractions, which they replace; at the same time it is frequently more expedient to use common fractions in certain operations, and, hence, they cannot be wholly dispensed with. Particular attention should be paid to the rules for multiplication and division—especially to the locating of the decimal point—and to the operations of changing a common fraction to a decimal and vice versa.

132. **Decimals** are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

133. The *denominator*, which is always ten or a multiple of ten, as 10, 100, 1,000, etc., is *not* expressed as it would be in common fractions, by writing it under the *numerator*, with a line between them; as, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure. In order to distinguish

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the unit figure, a period (.), called the **decimal point,** is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

134. The *reading* of a *decimal number* depends upon the *number* of *decimal places* in it, or the *number* of *figures* to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths.*

The second figure to the right of the unit figure expresses *hundredths.*

The third figure to the right of the unit figure expresses *thousandths.*

The fourth figure to the right of the unit figure expresses *ten-thousandths.*

The fifth figure to the right of the unit figure expresses *hundred-thousandths.*

The sixth figure to the right of the unit figure expresses *millionths.*

Thus:

.3 = $\frac{3}{10}$ = 3 tenths.

.03 = $\frac{3}{100}$ = 3 hundredths.

.003 = $\frac{3}{1000}$ = 3 thousandths.

.0003 = $\frac{3}{10000}$ = 3 ten-thousandths.

.00003 = $\frac{3}{100000}$ = 3 hundred-thousandths.

.000003 = $\frac{3}{1000000}$ = 3 millionths.

The first figure to the right of the unit figure is called the *first* **decimal place;** the second figure, the *second decimal place*, etc. We see in the above that the *number* of *decimal places* in a decimal equals the *number of ciphers* to the *right* of the figure 1 in the *denominator* of its *equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a *decimal point* is a whole number. The decimal point affects only the figures to its *right.*

When a *whole number* and *decimal* are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	. decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the *left* of the *decimal point* represent *whole numbers;* those to the *right* are *decimals.*

In *both* the decimals and whole numbers, the *units* place is made the *starting point* of notation and numeration. The *decimals decrease* on the scale of *ten* to the *right*, and the *whole* numbers *increase* on the scale of *ten* to the *left.* The *first* figure to the *left* of units is *tens*, and the *first* figure to the *right* of units is *tenths.* The *second* figure to the *left* of units is *hundreds*, and the *second* figure to the *right* is *hundredths.* The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on; the *whole* numbers on the *left* and the *decimals* on the *right.* The figures equally distant from units place correspond in name. The *decimals* have the ending *ths*, which distinguishes them from *whole* numbers. The following is the numeration of the number in the above table: Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred millionths.

The *decimals* increase to the *left*, on the scale of *ten*, the same as *whole* numbers; for, beginning at, say, 4-*thousandths*, in the table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

135. *Annexing* or *taking away* a *cipher* at the *right* of a *decimal* does *not* affect its value.

.5 is $\frac{5}{10}$; .50 is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, .5 = .50.

136. *Inserting* a *cipher* between a *decimal* and the *decimal point divides* the decimal by 10.

$.5 = \frac{5}{10}$; $\frac{5}{10} \div 10 = \frac{5}{100} = .05$.

137. *Taking away* a *cipher* from the *left* of a *decimal multiplies* the *decimal* by 10.

$.05 = \frac{5}{100}$; $\frac{5}{100} \times 10 = \frac{5}{10} = .5$.

138. In some cases it is convenient to express a mixed decimal fraction in the form of a common (improper) fraction. To do so it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for, $127.483 = 127\frac{483}{1000} = \frac{127000+483}{1000} = \frac{127483}{1000}$.

ADDITION OF DECIMALS.

139. The only respect in which *addition* of *decimals* differs from *addition* of *whole numbers*, is that while the unit figures are placed under each other in both cases, the right-hand figures are not necessarily in line when adding decimals.

Whole numbers begin at units and increase on the scale of 10, to the left. Decimals decrease on the scale of 10, to the right. Whole numbers are to the left of the decimal point and decimals are to the right of it. In whole numbers the *right-hand* side of a column of figures to be added, must be in line, and in decimals, the *left-hand* side must be in line, which brings the decimal points directly under each other.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

140. A decimal, as .342, ought really to be expressed as 0.342, but it is quite customary to omit the cipher on the left of the decimal point, though many authors use it.

EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—

	242.
	.36
	118.725
	1.005
	6.
	100.1
sum	468.190 Ans.

141. Rule.—*Place the numbers to be added so that the decimal points will be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

142. Find the sum of

(*a*)	.2143, .105, 2.3042, and 1.1417.	(*a*) 3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.	(*b*) 805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.	(*c*) 204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.	(*d*) 225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.	(*e*) 234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.	(*f*) 1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.	(*g*) 2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.	(*h*) 4,539.02625.

Ans.

SUBTRACTION OF DECIMALS.

143. For the same reason as in addition of decimals, the *left-hand* figures of *decimal numbers* are placed in line and the *decimal points* under each other.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend	.3063
subtrahend	.132
difference	.1743 Ans.

144. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

minuend	7.895
subtrahend	.725
difference	7.170 or 7.17 Ans.

145. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	11.000
subtrahend	.625
difference	10.375 Ans.

146. Rule.—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract, as in whole numbers, and place the decimal point in the remainder, directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE.

147. From

(*a*) 407.385 take 235.0004.
(*b*) 22.718 take 1.7042.
(*c*) 1,368.17 take 13.6817.
(*d*) 70.00017 take 7.000017.
(*e*) 630.630 take .6304.
(*f*) 421.73 take 217.162.
(*g*) 1.000014 take .00001.
(*h*) .783652 take .542314.

Ans.

(*a*) 172.3846.
(*b*) 21.0138.
(*c*) 1,354.4883.
(*d*) 63.000153.
(*e*) 629.9996.
(*f*) 204.568.
(*g*) 1.000004.
(*h*) .241338.

MULTIPLICATION OF DECIMALS.

148. In multiplication of decimals, we do not place the decimal points directly under each other as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.825
multiplier	13
	2475
	825
product	10.725 Ans.

In this example there are three decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

149. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	426
multiplier	.005
product	2.130 or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

150. It is *not* necessary to multiply by the ciphers on the *left* of a *decimal;* they merely determine the number of decimal places. Ciphers to the *right* of a decimal should be removed, as they only make more figures to deal with, and do not change the value.

151. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—

multiplicand	1.205
multiplier	1.15
	6025
	1205
	1205
product	1.38575 Ans.

In this example there are 3 decimal places in the multiplicand, and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

152. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

multiplicand	.232
multiplier	.001
product	.000232 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in each, the multiplier and the multiplicand, we must prefix 3 ciphers to the 232, to make 3 + 3, or 6, decimal places in the product.

153. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

154. Find the product of

(*a*) .000492 × 4.1418.
(*b*) 4,003.2 × 1.2.
(*c*) 78.6531 × 1.03.
(*d*) .3685 × .042.
(*e*) 178,352 × .01.
(*f*) .00045 × .0045.
(*g*) .714 × .00002.
(*h*) .00004 × .008.

Ans. (*a*) .0020377656. (*b*) 4,803.84. (*c*) 81.012693. (*d*) .015477. (*e*) 1,783.52. (*f*) .000002025. (*g*) .00001428. (*h*) .00000032.

DIVISION OF DECIMALS.

155. In division of decimals we pay *no* attention to the decimal point until *after* the division is performed. The *number of decimal places* in the *dividend must equal* (*be made to equal by annexing ciphers*) *the number of decimal places in the divisor*. Divide exactly as in whole numbers. *Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.*

EXAMPLE.—Divide .625 by 25.

SOLUTION.—

divisor dividend quotient

25) .625 (.025 Ans.
50

125
125
---
remainder 0

In this example there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

156. EXAMPLE.—Divide 6.035 by .05.

SOLUTION.—

divisor dividend quotient

.05) 6.035 (120.7 Ans.
5

10
10

35
35
---
remainder 0

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, one decimal place is pointed off in the quotient.

157. EXAMPLE.—Divide .125 by .005.

SOLUTION.—

divisor dividend quotient

.005) .125 (25 Ans.
10

25
25
---
remainder 0

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

158. EXAMPLE.—Divide 326 by .25.

divisor dividend quotient

SOLUTION.— .25)326.00(1304 Ans.

25

76

75

100

100

remainder 0

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

159. EXAMPLE.—Divide .0025 by 1.25.

SOLUTION.— 1.25).00250(.002 Ans.

250

remainder 0

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is clearly evident that the dividend 25 will not contain the divisor 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making 4+1, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off 5 − 2, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

160. Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there be a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still be a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried further.*

161. EXAMPLE.—What is the quotient of 199 divided by 15?

SOLUTION.— 15)199(13+$\frac{4}{15}$ Ans.

15

49

45

remainder 4

Or, 15)199.000(13.266+ Ans.

15

49

45

40

30

100

90

100

90

remainder 10

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

162. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place further. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that

the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667—. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247 +; if it was desired to retain five decimal places, it would be expressed as .24713—. Both the + and — signs are frequently omitted; they are seldom used outside of Arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

163. Divide

(*a*) 101.6688 by 2.36.
(*b*) 187.12264 by 123.107.
(*c*) .08 by .008.
(*d*) .0003 by 3.75.
(*e*) .0144 by .024.
(*f*) .00375 by 1.25.
(*g*) .004 by 400.
(*h*) .4 by .008.

Ans. (*a*) 43.08. (*b*) 1.52. (*c*) 10. (*d*) .00008. (*e*) .6. (*f*) .003. (*g*) .00001. (*h*) 50.

TO REDUCE A FRACTION TO A DECIMAL.

164. EXAMPLE.—$\frac{3}{4}$ equals what decimal?

SOLUTION.—
$$4\overline{)3.00}\ \underline{.75}$$
or $\frac{3}{4} = .75$. **Ans.**

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—
$$\begin{array}{r} 8\overline{)7.000}(.875 \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$
or $\frac{7}{8} = .875$. **Ans.**

165. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

166. Reduce the following common fractions to decimals:

(*a*) $\frac{15}{32}$. (*b*) $\frac{7}{8}$. (*c*) $\frac{21}{32}$. (*d*) $\frac{51}{64}$. (*e*) $\frac{4}{25}$. (*f*) $\frac{5}{8}$. (*g*) $\frac{10}{200}$. (*h*) $\frac{4}{1000}$.

Ans. (*a*) .46875. (*b*) .875. (*c*) .65625. (*d*) .796875. (*e*) .16. (*f*) .625. (*g*) .05. (*h*) .004.

---

167. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$ or $\frac{9}{12}$ of a foot. This, reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

$$\begin{array}{r} 12\overline{)9.00}(.75 \text{ of a foot.} \quad \textbf{Ans.} \\ \underline{84} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

168. Rule I.—*To reduce inches to decimal parts of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say, the fourth decimal place, carry the division one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1. Omit the signs + and —.*

EXAMPLES FOR PRACTICE.

169. Reduce to the decimal part of a foot:

(*a*) 3 in.
(*b*) $4\frac{1}{2}$ in.
(*c*) 5 in.
(*d*) $6\frac{5}{8}$ in.
(*e*) 11 in.

Ans. (*a*) .25. (*b*) .375. (*c*) .4167. (*d*) .5521. (*e*) .9167.

TO REDUCE A DECIMAL TO A FRACTION.

170. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

171. Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

172. Reduce the following to common fractions:

(*a*) .125.
(*b*) .625.
(*c*) .3125.
(*d*) .04.
(*e*) .06.
(*f*) .75.
(*g*) .15625.
(*h*) .875.

Ans. (*a*) $\frac{1}{8}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{5}{16}$. (*d*) $\frac{1}{25}$. (*e*) $\frac{3}{50}$. (*f*) $\frac{3}{4}$. (*g*) $\frac{5}{32}$. (*h*) $\frac{7}{8}$.

173. To express a decimal approximately as a fraction having a given denominator:

174. EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

175. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required*

EXAMPLES FOR PRACTICE.

176. Express

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans. (*a*) $\frac{5}{8}$. (*b*) $\frac{5}{16}$. (*c*) $\frac{5}{32}$. (*d*) $\frac{49}{64}$. (*e*) $\frac{39}{48}$. (*f*) $\frac{89}{96}$.

177. The sign for dollars is $. It is read dollars. $25 is read 25 dollars.

Since there are 100 cents in a dollar, one cent is 1-one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents mills.

Thus, $25.16 is read twenty-five dollars and sixteen cents; $25.168 is read twenty-five dollars, sixteen cents and eight mills.

178. The **vinculum**——, **parenthesis** ( ), **bracket** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8-3}$, or $13 \times (8-3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13 \times (8-3) = 13 \times 5 = 65. \quad \text{Ans.}$$
$$13 \times \overline{8-3} = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

179. In any series of numbers connected by the signs +, −, ×, and ÷, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction, until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

180. EXAMPLE.—What is the value of the following expression: $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} = ?$

SOLUTION.—$1,296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication *follows* 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* of the multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1{,}296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occurs, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $\overline{24 \times 3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

181. Find the values of the following expressions:

(*a*) $(8 + 5 - 1) \div 4$.
(*b*) $5 \times 24 - 32$.
(*c*) $5 \times 24 \div 15$.
(*d*) $144 - 5 \times 24$.
(*e*) $(1{,}691 - 540 + 559) \div 3 \times 57$.
(*f*) $2{,}080 + 120 - 80 \times 4 - 1{,}670$.
(*g*) $\overline{(90 + 60 \div 25)} \times 5 - 29$.
(*h*) $\overline{90 + 60} \div 25 \times 5$.

Ans. (*a*) 3. (*b*) 88. (*c*) 8. (*d*) 24. (*e*) 10. (*f*) 210. (*g*) 1. (*h*) 1.2.

ARITHMETIC.

(SECTION 4.)

PERCENTAGE.

182. **Percentage** is the process of calculating by *hundredths.*

183. The *term* **per cent.** is an abbreviation of the Latin words *per centum*, which mean *by the hundred.* A certain per cent. of a number is the number of hundredths of that number which is indicated by the number of units in the per cent. Thus, 6 per cent. of 125 is $125 \times \frac{6}{100} = 7.5$; 25 per cent. of 80 is $80 \times \frac{25}{100} = 20$; 43 per cent. of 432 pounds is $432 \times \frac{43}{100} = 185.76$ pounds.

184. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read *six per cent.;* $12\frac{1}{2}\%$ is read *twelve and one-half per cent.*, etc.

When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%.....	.01	$\frac{1}{100}$	150 %....	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%.....	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500 %....	5.00	$\frac{500}{100}$ or 5
5%.....	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}$%....	.0025	$\frac{1}{100}$ or $\frac{1}{400}$
10%.....	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}$%....	.005	$\frac{1}{100}$ or $\frac{1}{200}$
25%.....	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}$%....	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%.....	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}$%....	.08$\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%.....	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}$%....	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%.....	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}$%....	.16$\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%.....	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}$%....	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

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185. The names of the different elements used in percentage are: the *base*, the *rate per cent.*, the *percentage*, the *amount*, and the *difference.*

186. The **base** is the number on which the per cent. is computed.

187. The **rate** is the number of hundredths of the base to be taken.

188. The **percentage** is the part, or number of *hundredths*, of the base indicated by the rate; or, the percentage is the result obtained by multiplying the base by the rate.

Thus, when it is stated that 7% of $25 is $1.75, $25 is the base, 7% is the rate, and $1.75 is the percentage.

189. The **amount** is the sum of the base and percentage.

190. The **difference** is the remainder obtained by subtracting the percentage from the base.

Thus, if a man has $180, and he earns 6% more, he will have, altogether, $\$180 + \$180 \times .06$, or $\$180 + \$10.80 = \$190.80$. Here $180 is the base; 6%, the rate; $10.80, the percentage, and $190.80, the *amount.*

Again, if an engine of 125 horsepower uses 16% of it in overcoming friction and other resistances, the amount left for obtaining useful work is $125 - 125 \times .16 = 125 - 20 = 105$ horsepower. Here 125 is the base; 16%, the rate; 20, the percentage, and 105, the *difference.*

191. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate expressed decimally.*

EXAMPLE.—Out of a lot of 300 bushels of apples, 76% were sold. How many bushels were sold?

SOLUTION.—76%, the rate, expressed decimally, is .76; the base is 300; hence, the number of bushels sold, or the percentage, is by the above rule,

$300 \times .76 = 228$ bushels. Ans.

Expressing the rule as a

Formula, *percentage* = *base* × *rate.*

192. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then, 1%, or $\frac{1}{100}$, of the number, is $12 \div 6$, or 2. Consequently, if $2 = 1\%$, or $\frac{1}{100}$, 100%, or $\frac{100}{100}$, $= 2 \times 100 = 200$. But, since the same result may be arrived at by dividing 12 by .06, for $12 \div .06 = 200$, it follows that

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate, expressed decimally.*

Formula, *base* = *percentage* ÷ *rate.*

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If I sold 228 bushels, how many bushels did I buy?

SOLUTION.—Here 228 is the percentage, and 76%, or .76, is the rate; hence, applying the rule,

$228 \div .76 = 300$ bushels. Ans.

193. When the base and percentage are given, to find the rate, the rate may be found, expressed decimally, by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is $200 \times .01 = 2$. Now, if 1% is 2, 12 is evidently as many per cent. as the number of times that 2 is contained in 12, or $12 \div 2 = 6\%$. But the same result may be obtained by dividing 12, the percentage. by 200, the base, since $12 \div 200 = .06 = 6\%$. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate. expressed decimally.*

Formula, *rate* = *percentage* ÷ *base.*

EXAMPLE.—Bought 300 bushels of apples and sold 228 bushels. What per cent. of the total number of bushels was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule, rate $= 228 \div 300 = .76 = 76\%$. Ans.

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base and 25 is the percentage; hence, applying rule, $25 \div 875 = .02\frac{4}{7} = 2\frac{4}{7}\%$. Ans.

PROOF.—$875 \times .02\frac{4}{7} = 25$.

EXAMPLES FOR PRACTICE.

194. What per cent. of

(*a*) 360 is 90?
(*b*) 900 is 360?
(*c*) 125 is 25?
(*d*) 150 is 750?
(*e*) 280 is 112?
(*f*) 400 is 200?
(*g*) 47 is 94?
(*h*) 500 is 250?

Ans. (*a*) 25%. (*b*) 40%. (*c*) 20%. (*d*) 500%. (*e*) 40%. (*f*) 50%. (*g*) 200%. (*h*) 50%.

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195. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate, expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and 6% is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **189**, the amount is $200 + 12 = 212$. But the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate, expressed decimally.*

Formula, *amount* = *base* × (1 + *rate*).

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and 10% is the rate, and the amount is required. Hence, applying the rule,

$725 \times 1.10 = \$797.50$. Ans.

196. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate, expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **190**, the difference $= 200 - 12 = 188$. But the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate, expressed decimally.*

Formula, *difference* = *base* × (1 − *rate*).

EXAMPLE.—Bought 300 bushels of apples, and sold all but 24% of them. How many bushels were sold?

SOLUTION.—Here 300 is the base, 24% is the rate, and it is desired to find the difference. Hence, applying the rule,

$300 \times (1 - .24) = 228$ bushels. Ans.

197. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number increased by 6% of itself. Then it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200$ = the base. But the same result may be obtained by dividing 212 by $1 + .06$, or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate, expressed decimally.*

Formula, *base* = *amount* ÷ (1 + *rate*).

EXAMPLE.—The theoretical discharge of a certain pump, when running at a piston speed of 100 feet per minute, is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount; 25% is the rate, and, applying rule,

actual discharge $= 278{,}910 \div 1.25 = 223{,}128$ gallons. Ans.

198. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate, expressed decimally.*

Formula, *base* = *difference* ÷ (1 − *rate*).

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If there were 72 bushels left unsold, how many bushels did I buy?

SOLUTION.—Here 72 is the difference and 76% is the rate. Applying rule,

$$72 \div (1 - .76) = 300 \text{ bushels.} \quad \text{Ans.}$$

EXAMPLE.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference, and 30% is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds.} \quad \text{Ans.}$$

199. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **193,**

$$\text{gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \quad \text{Ans.}$$

200. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent., loss of itself) and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **193,**

$$\text{loss per cent.} = 1.8 \div 3 = .60 = 60\%. \quad \text{Ans.}$$

201. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and final values; divide this difference by the initial value.*

EXAMPLE.—If a man buys a house for $1,860, and some time afterwards builds a barn for 25% of the cost of the house, does he gain or lose, and how much per cent., if he sells both house and barn for $2,100?

SOLUTION.—The cost of the barn was $\$1{,}860 \times .25 = \$465$; consequently, the initial value, or cost, was $\$1{,}860 + \$465 = \$2{,}325$. Since he sold them for $2,100, he lost $\$2{,}325 - \$2{,}100 = \$225$. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss.} \quad \textbf{Ans.}$$

EXAMPLES FOR PRACTICE.

202. Solve the following:

		Ans.	
(*a*)	What is $12\frac{1}{2}\%$ of \$900?	(*a*)	\$112.50.
(*b*)	What is $\frac{4}{5}\%$ of 627?	(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}\%$ of 54?	(*c*)	18.
(*d*)	101 is $68\frac{3}{4}\%$ of what number?	(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}\%$ of what number?	(*e*)	940.8.
(*f*)	What % of 960 is 160?	(*f*)	$16\frac{2}{3}\%$.
(*g*)	What % of \$3,606 is $\$450\frac{3}{4}$?	(*g*)	$12\frac{1}{2}\%$.
(*h*)	What % of 280 is 112?	(*h*)	40%.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved? Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased $6\frac{1}{2}\%$, how many revolutions per minute would it then make? Ans. 134.19 revolutions.

3. The list price of a lot of silk goods is \$1,400; of some laces, \$1,150, and of some calico, \$340. If 25% discount was allowed on silk, 22% on the laces, and $12\frac{1}{2}\%$ on the calico, what was the actual cost of the purchase? Ans. \$2,244.50.

4. If I lend a man \$1,100, and this is $18\frac{1}{2}\%$ of the amount that I have on interest, how much money have I on interest? Ans. \$5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H.P.

6. By adding a condenser to a steam engine, the power was increased 14%, and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower, and required $3\frac{1}{2}$ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour, with the condenser, assuming the engine to run full power? Ans. 159.6 pounds.

DENOMINATE NUMBERS.

203. A **denominate number** is a concrete number, and may be either simple or compound, as 8 quarts, 5 feet, 10 inches, etc.

204. A **simple denominate number** consists of units of but one denomination, as 16 cents, 10 hours, 5 dollars, etc.

205. A **compound denominate number** consists of units of two or more denominations of a similar kind, as 3 yards 2 feet 1 inch ; 34 square feet 57 square inches.

206. In **whole numbers** and in **decimals** the *law* of increase and decrease is on the scale of 10, but in **compound** or **denominate numbers** the scale varies.

MEASURES.

207. A **measure** is a *standard unit*, established by *law* or *custom*, by which *quantity* of any kind is measured. The *standard unit* of **dry measure** is the Winchester bushel ; of **weight,** the pound ; of **liquid measure,** the gallon, etc.

208. Measures are of six kinds :

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

209. Measures of extension are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE 3.

		Abbreviation.	in.	ft.	yd.	rd.	fur.	mi.
12	inches (in.) = 1 foot	ft.						
3	feet = 1 yard	yd.	36 =	3 =	1			
5.5	yards = 1 rod	rd.	198 =	16½ =	5.5 =	1		
40	rods = 1 furlong	fur.	7,920 =	660 =	220 =	40 =	1	
8	furlongs = 1 mile	mi.	63,360 =	5,280 =	1,760 =	320 =	8 =	1

SURVEYOR'S LINEAR MEASURE.

TABLE 4.

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods, 100 links	= 1 chain	ch.
80 chains	= 1 mile	mi.

mi.	ch.	rd.	li.	in.
1 =	80 =	320 =	8,000 =	63,360

210. The linear unit, generally used by surveyors, is **Gunter's chain,** which is equal to 4 rods, or 66 feet.

211. An **engineer's chain,** used by civil engineers, is 100 feet long, and consists of 100 links. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE 5.

144	square inches (sq. in.)	= 1 square foot	sq.ft.
9	square feet	= 1 square yard	sq.yd.
30¼	square yards	= 1 square rod	sq.rd.
160	square rods	= 1 acre	A.
640	acres	= 1 square mile	sq.mi.

sq.mi.	A.	sq.rd.	sq.yd.	sq.ft.	sq.in.
1 =	640 =	102,400 =	3,097,600 =	27,878,400 =	4,014,489,600

SURVEYOR'S SQUARE MEASURE.

TABLE 6.

625	square links	= 1 square rod	sq.rd.
16	square rods	= 1 square chain	sq.ch.
10	square chains	= 1 acre	A.
640	acres	= 1 square mile	sq.mi.
36	square miles (6 mi. square)	= 1 township	Tp.

sq.mi.	A.	sq.ch.	sq.rd.	sq.li.
1 =	640 =	6,400 =	102,400 =	64,000,000

CUBIC MEASURE.

TABLE 7.

1728	cubic inches (cu. in.)	= 1 cubic foot	cu.ft.
27	cubic feet	= 1 cubic yard	cu.yd.
128	cubic feet	= 1 cord	cd.
24¾	cubic feet	= 1 perch	P.

cu.yd.	cu.ft.	cu.in.
1 =	27 =	46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE 8.

16	ounces (oz.)	= 1 pound	lb.
100	pounds	= 1 hundredweight	cwt.
20 cwt., or 2,000	lb.	= 1 ton	T.

T.	cwt.	lb.	oz.
1 =	20 =	2,000 =	32,000

212. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles. One avoirdupois pound contains 7,000 grains.

LONG TON TABLE.

TABLE 9.

16 ounces	=	1 pound	lb.
112 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,240 lb.	=	1 ton	T.

213. In all the calculations throughout this and the succeeding volumes, 2,000 pounds will be considered one ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE 10.

24 grains (gr.)	=	1 pennyweight	pwt.
20 pennyweights	=	1 ounce	oz.
12 ounces	=	1 pound	lb.

lb.	oz.	pwt.	gr.
1	= 12	= 240	= 5,760

214. Troy weight is used in weighing gold and silverware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE 11.

4 gills (gi.)	=	1 pint	pt.
2 pints	=	1 quart	qt
4 quarts	=	1 gallon	gal.
31½ gallons	=	1 barrel	bbl.
2 barrels, or 63 gallons	=	1 hogshead	hhd.

hhd.	bbl.	gal.	qt.	pt.	gi.
1 =	2 =	63 =	252 =	504 =	2,016

DRY MEASURE.

TABLE 12.

2 pints (pt.)	=	1 quart	qt.
8 quarts	=	1 peck	pk.
4 pecks	=	1 bushel	bu.

bu.	pk.	qt.	pt.
1 =	4 =	32 =	64

MEASURE OF TIME.

TABLE 13.

60 seconds (sec.)	=	1 minute	min.
60 minutes	=	1 hour	hr.
24 hours	=	1 day	da.
7 days	=	1 week	wk.
365 days / 12 months	=	1 common year	yr.
366 days	=	1 leap year.	
100 years	=	1 century.	

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE 14.

60 seconds (″)	=	1 minute	′.
60 minutes	=	1 degree	°.
90 degrees	=	1 right angle or quadrant	∟.
360 degrees	=	1 circle	cir.

cir.
1 = 360° = 21,600′ = 1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE 15.

10 mills (m.)	=	1 cent	ct.
10 cents	=	1 dime	d.
10 dimes	=	1 dollar	$.
10 dollars	=	1 eagle	E.

E.	$	d.	ct.	m.
1 =	10 =	100 =	1,000 =	10,000

MISCELLANEOUS TABLE.

TABLE 16.

12 things are 1 dozen.	1 meter is nearly 39.37 inches.
12 dozen are 1 gross.	1 hand is 4 inches.
12 gross are 1 great gross.	1 palm is 3 inches.
2 things are 1 pair.	1 span is 9 inches.
20 things are 1 score.	24 sheets are 1 quire.
1 league is 3 miles.	20 quires, or 480 sheets, are 1 ream.
1 fathom is 6 feet.	1 bushel contains 2,150.4 cubic in.

1 U. S. standard gallon (also called a wine gallon) contains 231 cubic in.
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all of the above tables.

REDUCTION OF DENOMINATE NUMBERS.

215. Reduction of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination or from a lower to a higher—either is reduction. As,

2 hours = 120 minutes.
32 ounces = 2 pounds.

216. Principle.—Denominate numbers are changed to *lower* denominations by *multiplying*, and to *higher* denominations by *dividing*.

To reduce denominate numbers to lower denominations:

217. EXAMPLE.—Reduce 5 yd. 2 ft. 7 in. to inches.

SOLUTION.—

yd.	ft.	in.
5	2	7
3		
15 ft.		
2 ft.		
17 ft.		
12		
34		
17		
204 in.		
7 in.		
211 inches. Ans.		

EXPLANATION.—Since there are 3 feet in 1 yard, in 5 yards there are 5 × 3, or 15 feet, and 15 feet plus 2 feet = 17 feet. There are 12 inches in a foot; therefore, 12 × 17 = 204 inches, and 204 inches plus 7 inches = 211 inches = number of inches in 5 yards 2 feet and 7 inches. Ans.

218. EXAMPLE.—Reduce 6 hours to seconds.

SOLUTION.—

6	hours.
60	
360	minutes.
60	
21600 seconds. Ans.	

EXPLANATION.—As there are 60 minutes in one hour, in six hours there are 6 × 60, or 360 minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

219. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet; as,

rd.	yd.	ft.	in.
3	0	0	6

220. Rule.—*Multiply the number representing the highest denomination by the number of units in the next lower required to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

221. Reduce

		Ans.	
(*a*)	4 rd. 2 yd. 2 ft. to ft.	(*a*)	74 ft.
(*b*)	4 bu. 3 pk. 2 qt. to qt.	(*b*)	154 qt.
(*c*)	13 rd. 5 yd. 2 ft. to ft.	(*c*)	231.5 ft.
(*d*)	5 mi. 100 rd. 10 ft. to ft.	(*d*)	28,060 ft.
(*e*)	8 lb. 4 oz. 6 pwt. to gr.	(*e*)	48,144 gr.
(*f*)	52 hhd. 24 gal. 1 pt. to pt.	(*f*)	26,401 pt.
(*g*)	5 cir. 16° 20′ to minutes.	(*g*)	108,980′.
(*h*)	14 bu. to qt.	(*h*)	448 qt.

To reduce lower to higher denominations:

222. EXAMPLE.—Reduce 211 in. to higher denominations.

SOLUTION.—

12) 211 in.
3) 17 ft. + 7 in.
5 yd. + 2 ft. Ans.

EXPLANATION.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There

are 3 feet in 1 yard ; therefore, 17 feet divided by 3 = 5 yards and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

223. EXAMPLE.—Reduce 15,735 grains Troy weight to higher denominations.

SOLUTION.—

```
24)15735 gr. (655 pwt.
   144
   ---
    133
    120
    ---
     135
     120
     ---
      15 gr.
20)655 pwt. (32 oz.
  60
  --
   55
   40
   --
   15 pwt.
12)32 oz. (2 lb.
  24
  --
   8 oz.
```

EXPLANATION.—There are 24 grains in 1 pennyweight, and in 15,735 grains there are as many pennyweights as 24 is contained in 15,735, or 655 pennyweights and 15 grains remaining. There are 20 pennyweights in 1 ounce, and in 655 pennyweights there are 32 ounces and 15 pennyweights remaining. There are 12 ounces in 1 pound, and in 32 ounces there are 2 pounds and 8 ounces remaining. The last quotient and the three remainders constitute the answer, 2 pounds 8 ounces 15 pennyweights 15 grains.

The above problem is worked out by long division, because the numbers are too large to solve easily by short division. The student may use either method.

224. Rule.—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by the number of units of its denomination required to make one unit of the next higher. Continue until the highest denomination is reached, or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.*

EXAMPLES FOR PRACTICE.

225. Reduce to units of higher denominations :

(*a*) 7,460 sq. in. ; (*b*) 7,580 sq. yd. ; (*c*) 148,760 cu. in. ; (*d*) 7,896 cu. ft. to cd. ; (*e*) 17,651″ ; (*f*) 1,120 cu. ft. to cd. ; (*g*) 8,000 gi. ; (*h*) 36,450 lb.

Ans. (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
(*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in
(*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
(*d*) 61 cd. 88 cu. ft.
(*e*) 4° 54′ 11″.
(*f*) 8 cd. 96 cu. ft.
(*g*) 3 hhd. 61 gal.
(*h*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

226. EXAMPLE.—Find the sum of 3 cwt. 46 lb. 12 oz. ; 8 cwt. 12 lb 13 oz. ; 12 cwt. 50 lb. 13 oz. ; 27 lb. 4 oz.

SOLUTION.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

EXPLANATION.—Begin to add at the right-hand column : 4 + 13 + 13 + 12 = 42 ounces ; as 16 ounces make 1 pound, 42 ounces ÷ 16 = 2 and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under ounce column, and add 2 pounds to the next or pound column. Then, 2 + 27 + 50 + 12 + 46 = 137 pounds ; as 100 pounds make a hundredweight, 137 ÷ 100 = 1 hundredweight and a remainder of 37 pounds. Place the 37 under the pounds column, and add 1 hundredweight to the next or hundredweight column. Next, 1 + 12 + 8 + 3 = 24 hundredweight.

20 hundredweight make a ton; therefore $24 \div 20 = 1$ ton and 4 hundredweight remaining. Hence, the sum is 1 ton 4 hundredweight 37 pounds 10 ounces. Ans.

227. EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

SOLUTION.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	$3\frac{1}{2}$	0	2	
or	26	3	1	8	Ans.

EXPLANATION.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the next column plus 2 yards = 9 yards, or $9 \div 5\frac{1}{2} = 1$ rod and $3\frac{1}{2}$ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the $\frac{1}{2}$ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches. Ans.

228. EXAMPLE.—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

SOLUTION.—When 47 ft. is reduced it equals 2 rd. 4 yd. 2 ft., which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	$1\frac{1}{2}$	1	10	
or	6	2	0	4	Ans.

229. Rule.—*Place the numbers so that like denominations are under each other. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added, and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

230. What is the sum of

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{2}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{5}{8}$ lb.?

(*d*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 tons 11 cwt. 49 lb. 14 oz.; 16 tons 47 lb. 13 oz.; 20 tons 13 cwt. 14 lb. 6 oz.; 11 tons 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans.
(*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 tons 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

231. EXAMPLE.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in., take 9 rd. 4 yd. $10\frac{1}{4}$ in.

SOLUTION.—

rd.	yd.	ft.	in.	
21	2	2	$6\frac{1}{2}$	
9	4	0	$10\frac{1}{4}$	
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$	Ans.

EXPLANATION.—Since $10\frac{1}{4}$ inches cannot be taken from $6\frac{1}{2}$ inches, we must borrow 1 foot, or 12 inches, from the 2 feet in the next column and add it to the $6\frac{1}{2}$. $6\frac{1}{2} + 12 = 18\frac{1}{2}$. $18\frac{1}{2}$ inches $- 10\frac{1}{4}$ inches $= 8\frac{1}{4}$ inches. Then, 0 foot from the 1 remaining foot = 1 foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or $5\frac{1}{2}$ yards, from 21 rods and add it to 2. $2 + 5\frac{1}{2} = 7\frac{1}{2}$; $7\frac{1}{2} - 4 = 3\frac{1}{2}$ yards. 9 rods from 20 rods = 11 rods. Hence, the remainder is 11 rods $3\frac{1}{2}$ yards 1 foot $8\frac{1}{4}$ inches. Ans.

To avoid fractions as much as possible, we reduce the $\frac{1}{2}$ yard to inches, obtaining 18 inches; this added to $8\frac{1}{4}$ inches, gives $26\frac{1}{4}$ inches, which equals 2 feet $2\frac{1}{4}$ inches. Then, 2 feet + 1 foot = 3 feet = 1 yard, and 3 yards + 1 yard = 4 yards. Hence, the above answer becomes 11 rods 4 yards 0 feet $2\frac{1}{4}$ inches.

232. EXAMPLE.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft. ?

SOLUTION.—47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	$3\frac{1}{2}$	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

233. EXAMPLE.—How many years, months, days, and hours between 4 o'clock P.M. of June 15, 1868, and 10 o'clock A.M., September 28, 1891 ?

SOLUTION.—

yr.	mo.	da.	hr.	
1891	8	28	10	
1868	5	15	16	
23	3	12	18	Ans.

EXPLANATION.—Counting 24 hours in 1 day, 4 o'clock P.M. is the 16th hour from the beginning of the day, or midnight. On September 28, 8 months and 28 days have elapsed, and on June 15, 5 months and 15 days. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as **1** month.

234. Rule.—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be borrowed from the minuend of the next higher denomination, reduced and added to it.*

EXAMPLES FOR PRACTICE.

235. From

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.
(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.
(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.

(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.
(*f*) 14 mi. 34 rd. 16 yd. 13 ft. 11 in. take 3 mi. 27 rd. 11 yd. 4 ft. 10 in.

Ans.
(*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

236. EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

SOLUTION.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

EXPLANATION.—15 grains × 12 = 180 grains. 180 ÷ 24 = 7 pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next. Now, 13 × 12 + 7 = 163 pennyweights ; 163 ÷ 20 = 8 ounces and 3 pennyweights remaining. Then, 5 × 12 + 8 = 68 ounces; 68 ÷ 12 = 5 pounds and 8 ounces remaining. Then, 7 × 12 + 5 = 89 pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains. Ans.

237. Rule.—*Multiply the number representing each denomination by the multiplier, and reduce each product to the next higher denomination, writing the remainders under each denomination, and carrying the quotient to the next, as in Addition of Denominate Numbers.*

238. NOTE. — In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the above example, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. 43,047 × 1.2 = 51,656.4 gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, 43,047 × 12 = 516,564 gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as above. The student may use either method.

EXAMPLES FOR PRACTICE.

239. Multiply

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 4 wk. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans. (*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 116 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

240. EXAMPLE.—Divide 48 lb. 11 oz. 6 pwt. by 8.

SOLUTION.—

	lb.	oz.	pwt.	gr.
8)	48	11	6	0
	6 lb.	1 oz.	8 pwt.	6 gr. Ans.

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 ounces once with 3 ounces remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pennyweights; 66 pennyweights $\div 8 = 8$ pennyweights and 2 remaining; 2×24 grains $= 48$ grains; 48 grains $\div 8 = 6$ grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains. Ans.

EXAMPLE.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

SOLUTION.—

	lb.	oz.	pwt.
6)	2	8	10
		5 oz.	8 pwt. 8 gr. Ans.

EXPLANATION.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds $= 24$ ounces, and 24 ounces $+ 8$ ounces $= 32$ ounces; 32 ounces $\div 6 = 5$ ounces and 2 ounces over. 2 ounces $= 40$ pennyweights. 40 pennyweights $+ 10$ pennyweights $= 50$ pennyweights, and 50 pennyweights $\div 6 = 8$ pennyweights and 2 pennyweights over. 2 pennyweights $= 48$ grains, and 48 grains $\div 6 = 8$ grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains. Ans.

241. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

SOLUTION.—

	rd.	yd.	ft.	rd.	yd.	ft.	in.	
112)	820	4	2	(7	1	2	5.143	Ans.

784

36 rd. rem.

5.5

180

180

198.0 yd.

4

112) 202 yd. (1 yd.

112

90 yd. rem.

3

270 ft.

2 ft.

112) 272 ft. (2 ft.

224

48 ft. rem.

12

96

48

112) 576 in. (5.1428 + in., or 5.143 in.

560

160

112

480

448

320

224

960

896

64

EXPLANATION.—The first quotient is 7 rods with 36 rods remaining. $5.5 \times 36 = 198$ yards; 198 yards $+ 4$ yards $=$ 202 yards; 202 yards $\div 112 = 1$ yard and 90 yards remaining. $90 \times 3 = 270$ feet; 270 feet $+ 2$ feet $= 272$ feet; 272 feet $\div 112 = 2$ feet and 48 feet remaining; $48 \times 12 = 576$ inches; 576 inches $\div 112 = 5.143$ inches, nearly. Ans.

The preceding example is solved by long division, because the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

242. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

243. Divide

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 tons 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans.
- (*a*) 17 mi. $41\frac{7}{11}$ rd.
- (*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
- (*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
- (*d*) 4 sq.yd. 4 sq.ft. $2\frac{5}{18}$ sq.in.
- (*e*) 12 mi. 112 rd. 2 yd.
- (*f*) 6 tons 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
- (*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
- (*h*) 1 mi. $38\frac{22}{25}$ rd.

ARITHMETIC.

(SECTION 5.)

INVOLUTION.

244. If a product consists of equal factors, it is called a **power** of one of those equal factors, and one of the equal factors is called a **root** of the product. The power and the root are named according to the number of equal factors in the product. Thus, 3×3, or 9, is the *second power*, or **square,** of 3; $3 \times 3 \times 3$, or 27, is the *third power*, or **cube,** of 3; $3 \times 3 \times 3 \times 3$, or 81, is the **fourth power** of 3. Also, 3 is the **second root,** or **square root,** of 9; 3 is the **third root,** or **cube root,** of 27; 3 is the **fourth root** of 81.

245. For the sake of brevity,

3×3 is written 3^2, and read **three square,**
or *three exponent two;*

$3 \times 3 \times 3$ is written 3^3, and read **three cube,**
or *three exponent three;*

$3 \times 3 \times 3 \times 3$ is written 3^4, and read **three fourth,**
or *three exponent four;*

and so on.

A number written above and to the right of another number, to show how often the latter number is used as a factor, is called an **exponent.** Thus, in 3^{12}, the number ^{12} is the exponent, and shows that 3 is to be used as a factor twelve times; so that 3^{12} is a contraction for

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3.$$

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In an expression like 3^5, the exponent ^{5} shows how often 3 is used as a factor. Hence, if the exponent of a number is unity, the number is used once as a factor ; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

246. If the side of a square contains 5 inches, the area of the square contains 5×5, or 5^2, square inches. If the edge of a cube contains 5 inches, the volume of the cube contains $5 \times 5 \times 5$, or 5^3, cubic inches. It is for this reason that 5^2 and 5^3 are called the square and cube of 5, respectively.

247. To find any power of a number:

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.—

$35 \times 35 \times 35$

or

```
  35
  35
 175
105
1225
  35
6125
3675
```

cube = 42875 Ans.

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.—

$15 \times 15 \times 15 \times 15$

or

```
  15
  15
  75
 15
 225
  15
1125
225
3375
  15
16875
3375
```

fourth power = 50625 Ans.

EXAMPLE 3.— 1.2^3 = what?

SOLUTION.—

$1.2 \times 1.2 \times 1.2$

or

```
  1.2
  1.2
 1.44
  1.2
 288
144
```

cube = 1.728 Ans.

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $\left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

248. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

Raise the following to the powers indicated:

(*a*) 85^2.
(*b*) $(\frac{12}{13})^2$.
(*c*) 6.5^2.
(*d*) 14^4.
(*e*) $(\frac{3}{4})^3$.
(*f*) $(\frac{5}{6})^3$.
(*g*) $(\frac{7}{2})^3$.
(*h*) 1.4^5.

Ans. (*a*) 7,225. (*b*) $\frac{144}{169}$. (*c*) 42.25. (*d*) 38,416. (*e*) $\frac{27}{64}$. (*f*) $\frac{125}{216}$. (*g*) $\frac{343}{8}$. (*h*) 5.37824.

EVOLUTION.

249. Evolution is the reverse of involution. It is the process of finding the root of a number that is considered as a power.

250. The **square root** of a number is that number which when used twice as a factor produces the number.

Thus, 2 is the square root of 4, since 2×2, or $2^2 = 4$.

251. The **cube root** of a number is that number which when used three times as a factor produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or $3^3 = 27$.

252. The **radical sign** $\surd$ when placed before a number indicates that some root of that number is to be found. The vinculum is almost always used in connection with the radical sign, as shown in Art. **253.**

253. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.

$\sqrt[3]{125}$ denotes the *cube root* of 125.

$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

254. When the square root is to be extracted, the index is generally omitted. Thus, $\surd 100$ indicates the square root of 100. Also, $\surd 225$ indicates the square root of 225.

255. In any number, the figures beginning with the first digit* at the left and ending with the last digit at the right, are called the **significant figures** of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

The part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

In speaking of the significant figures or of the significant part of a number, we consider the figures, in their proper order, from the first digit at the left to the last digit at the right, but we pay no attention to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3 and the same significant part 2103.

*A cipher is not a digit.

SQUARE ROOT.

256. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This is done by pointing off the number into *periods* of *two* figures each, *beginning at the right.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83′74′08′01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures, since there are (5′06′25) 3 periods. The extreme left-hand period may contain either one or two figures, according to the size of the number squared.

257. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$, $.13^2 = .0169$, $.751^2 = .564001$, etc.

258. It will also be noticed that the square of a decimal is always less than the decimal. Hence, the square root of a number wholly decimal is greater than the number itself. If it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to point off a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher to complete the period.

259. There are comparatively few numbers that can be separated into exactly equal factors; these numbers are called

perfect powers, and the factors are called *rational factors.* Numbers that cannot be separated into exactly equal factors are called **surds,** and the factors are called *irrational factors.* In the numbers from 1 to 1,000, inclusive, there are only 42 perfect powers, not counting 1; and of these only 30 are perfect squares and 9 perfect cubes.

The root of any number that cannot be divided into as many equal factors as there are units in the index of the root contains an interminable decimal. For example, the number 20 lies between 16 $(=4^2)$ and 25 $(=5^2)$; hence, the square root of 20, or $\sqrt{20}$, is greater than 4 and less than 5, and is therefore equal to 4 plus an interminable decimal. In other words, no matter to how many figures the square root of 20 may be calculated, the root will never be found exactly.

260. Although the root of a surd cannot be found exactly, as close an approximation may be obtained as is desired. In practice, five significant figures are all that are likely to be required, and four are generally sufficient. In the following examples, all roots will be calculated to five figures, unless the given number is a perfect power whose root contains less than five figures.

261. The student will find the following principles of value, both in connection with the extraction of roots and in other arithmetical calculations:

a. In general, if any two numbers are multiplied together —no matter how many significant figures they contain—the first five significant figures of the product will be the same as the first five significant figures of the product obtained by multiplying the same two numbers when limited to five significant figures.

For example, the product of 4,562,357 and 6,421,849 is 29,298,767,738,093; limiting the numbers to five significant figures, the product of 45,624 and 64,218 is 2,929,882,032; and the value of both these products to five significant figures is 29,299. In other words, if only five significant figures are required in the product, it is not

necessary to use more than five significant figures in the multiplier and multiplicand, the remaining figures, if any, being replaced by ciphers, and the fifth figures being increased by 1 if the sixth figure is 5 or a larger digit. In some cases, however, the fifth figure may be one unit too large or one unit too small; hence, if it is necessary that the fifth figure be absolutely exact, it is better to limit the multiplier and multiplicand to six figures instead of five.

For example, $4,562,347 \times 6,421,849 = 29,298,703,519,603$, or 29,299,000,000,000 to five significant figures; $4,562,300 \times 6,421,800 = 29,298,178,140,000 = 29,298,000,000,000$ to five significant figures, the fifth figure being 1 less than it should be; but $4,562,350 \times 6,421,850 = 29,298,727,347,500 = 29,299,000,000,000$ to five significant figures.

b. If the divisor and dividend are limited to six significant figures, the quotient will always be correct to five (usually to six) significant figures, regardless of how many significant figures there may have been in the dividend and divisor.

For example, $6,421,849 \div 4,562,357 = 1.407572+ = 1.4076$ to five significant figures; also, $642,185 \div 456,236 = 1.407571+ = 1.4076$ to five significant figures.

c. If the number whose root is to be extracted be limited to six significant figures, the root will be correct to five (usually to six) significant figures.

262. These principles may all be summed up in the following general statement: *In any series of arithmetical operations —addition, subtraction, multiplication, division, involution, and evolution—if it be desired to have the final result limited to a certain number of significant figures, it is unnecessary to use more significant figures in any of the numbers operated on than the desired number in the result plus 1.* For example, if only four significant figures are desired in the final result, all the numbers used in the various operations may be limited to $4 + 1 = 5$ significant figures, the fifth figure being increased by 1 in all cases if the sixth figure is 5 or a greater digit.

From the foregoing, it follows that any method that will give five significant figures of the root correctly will be

sufficiently exact for all practical purposes. Such a method will now be explained for extracting square root.

263. Suppose it is desired to find the square root of 20; that is, $\sqrt{20} = ?$ The problem is to divide 20 into two equal factors, or into two factors, the first five significant figures of which shall be equal. Since 20 is not a perfect square, inspection shows that one of the equal factors is 4 plus an interminable decimal, since 20 lies between $4^2 = 16$ and $5^2 = 25$. Dividing 20 by 4, the result is 5; i. e., $4 \times 5 = 20$. Now, by taking the average of these unequal factors, a new factor will be obtained, which will be nearer the correct value of the root than either of the two unequal factors, viz., $\frac{4+5}{2} = 4.5$, the square of which is $4.5^2 = 20.25$.

Assuming 4.5 for a new factor and dividing 20 by it, the result is $20 \div 4.5 = 4.444+$; that is, $4.444 \times 4.5 = 20$, nearly, the product not being exactly equal to 20 because 4.444 was used as one factor, instead of $4\frac{4}{9}$, the exact value. Again, taking the average of the two factors, $\frac{4.444 + 4.5}{2} = 4.472$, which is the root correct to at least three figures.

Assuming 4.47 to be one of the factors and dividing 20 by it to obtain the other, the result is $20 \div 4.47 = 4.474272+$; that is, $4.47 \times 4.474273 = 20$, very nearly. The average of these two factors is $\frac{4.47 + 4.474272}{2} = 4.472136+ = 4.4721$ to five significant figures. The exact root to 13 figures is 4.472135954999+.

That 4.4721 is the square root of 20 correct to five figures may easily be proved by squaring it; thus, $4.4721^2 = 19.99967841$, or 20.000 to five figures. Since the square agrees with the given number to five figures, the root is correct to five figures.

264. A close examination of the foregoing results reveals some remarkable facts. (1) The value of the first average 4.5 is correct to two figures of the root. (2) The value of the second average 4.472+ is correct to four figures of the root. (3) The value of the third average 4.472136 is correct

to seven figures of the root. (4) All these averages are somewhat greater than the correct value of the root. (5) Of the two factors used in finding the average, one is a little greater and the other a little less than the correct value of the root. (6) Each step of the process gives a result approaching more and more nearly to the correct value of the root.

265. Calling the first average value the **first approximation,** the second average value the **second approximation,** and the third average value the **third approximation,** the following general method of procedure may be adopted: *Calculate the first approximation to two significant figures; the second approximation to three significant figures; and the third approximation to five significant figures. It is not safe to calculate the second approximation to more than three significant figures, because the fourth figure cannot, as a rule, be depended on. If the second significant figure of the first approximation be determined correctly, the third approximation will always be correct to at least five significant figures.*

266. The method will now be applied to numbers in general, and the best manner of explaining it is by means of examples.

EXAMPLE.— $\sqrt{714,627} = ?$

SOLUTION.—The first step is to point off the number into periods of two figures each, obtaining 71'46'27. To find the first approximation, only the first two significant figures are necessary; in this case, the first period, 71. The first figure of the root is evidently 8, since $8^2 = 64$ and $9^2 = 81$. The two factors then are 8 and $71 \div 8 = 8.87+$. The first approximation is $\frac{8 + 8.87}{2} = 8.43+ = 8.4$ to two figures.

To find the second approximation, use the first two periods, or 7146, and drop the decimal point in the first approximation. One factor is then 84 and the other $7146 \div 84 = 85.07+$. The second approximation is therefore $\frac{84 + 85.07}{2} = 84.53+$, or 84.5 to three figures.

To find the third approximation, use the first three periods, or 714627, and drop the decimal point in the second approximation. One factor is then 845 and the other is $714627 \div 845 = 845.712+$. The third approximation is therefore $\frac{845 + 845.712}{2} = 845.356$, or 845.36 to five significant figures. Ans.

REMARK.—It will be noticed in the last example, and also in those that follow, that when finding the unknown factor to be used in determining the value of the first, second, or third approximation, the division is carried one place farther than the number of figures desired in the approximation and that no attention is paid to the succeeding figures. Thus, in the last example, $71 \div 8 = 8.875$, or 8.88, correct to three figures, while the number used was 8.87. The reason for this is that the value obtained for the approximation would be the same in either case, and it saves time to calculate as here shown. For instance, using 8.88 for the second factor, the first approximation is $\frac{8 + 8.88}{2}$ $= 8.44$, or 8.4 to two figures.

267. The decimal point is located by employing the following principle: *There must be as many figures in the integral* part of the root as there are periods in the integral part of the given number whose root is to be found. If the given number is wholly decimal and there are two or more ciphers between the decimal point and the first significant figure, there will be as many ciphers between the decimal point and the first significant figure of the root as there are entirely cipher periods between the decimal point and the first significant figure of the given number.* Had the number in the last example been 71.4627, the root would have been 8.4536; had it been .714627, the root would have been .84536; had it been .0000714627, the root would have been .0084536. In the latter case, the number would have been pointed off thus, .00′00′71′46′27.

268. In all cases, numbers having the same significant parts and the same number of significant figures in the first (or left-hand) period of the significant part of the number, have the same significant figures in the root, the roots differing only in the position of the decimal point.

EXAMPLE.— $\sqrt{714.627} = ?$

SOLUTION.—Pointing off into periods, we have 7′14.62′70, adding a cipher to complete the last period. In all cases when pointing off the decimal part of numbers, begin at the decimal point and point off to the right, and add ciphers to the last period when it does not contain enough figures to make up a period. Since the first period contains but one figure and it is necessary to have two figures at least in order that the first approximation may be correct to two figures, regard the

* The *integral* part of a number is the part to the left of the decimal point. Thus, the integral part of 1,726.943 is 1,726.

decimal point as situated between 7 and 1 instead of between 4 and 6, thus obtaining 7.1 for the first two figures of the given number. The first two figures of the square root of 7.1 will be the same as the first two figures of the square root of 714.

It is evident that the first figure of the root is 2, since $2^2 = 4$ and $3^2 = 9$. Using 2 as one factor, the other is $7.1 \div 2 = 3.55$, and the first approximation is $\frac{2 + 3.55}{2} = 2.77+$, or 2.8 to two figures. Had 3 been used as one factor, the other would have been $7.1 \div 3 = 2.36+$, and the first approximation would have been $\frac{3 + 2.36}{2} = 2.68$, or 2.7 to two figures. In the first case, the difference between the two factors is $3.55 - 2 = 1.55$; in the second case, the difference is $3 - 2.36 = .64$. As the factors are more nearly equal in the second case than in the first, it is evident that 2.7 is more nearly equal to the correct value of the root than 2.8 is; hence, 2.7 will be used for the first approximation.

For the second approximation, use the first two periods and 27 for one factor, the other factor being $715 \div 27 = 26.48+$; hence, the second approximation $= \frac{27 + 26.48}{2} = 26.74$, or 26.7 to three figures. We used 715 for the first three figures of the given number, instead of 714, because the fourth figure was 6 and the number correct to three figures was 715. In finding the third approximation, the first three periods may be used or all the figures; the result will be the same in either case. Since it is better to use six figures than five, move the decimal point two places to the right, obtaining 71462.7; one factor is 267 and the other $71462.7 \div 267 = 267.650+$. The third approximation is $\frac{267 + 267.650}{2} = 267.325$, or 267.33 to five figures. Since there are two periods in the integral part of the given number, there are two figures in the integral part of the root, and $\sqrt{714.627} = 26.733$. Ans.

269. When determining the first approximation, that number should always be used for the first factor which will make the less difference between it and the second factor, as was done in the last example. Thus, for 2.5, the factors would be 2 and 1.25, the difference between them being .75. If 1 were selected for the first factor, the other would be $2.5 \div 1 = 2.5$, and the difference between them 1.5. In one case, the first approximation would be $\frac{2 + 1.25}{2} = 1.6+$, and in the other case, $\frac{1 + 2.5}{2} = 1.8-$. Since $1.6^2 = 2.56$ and

$1.8^2 = 3.24$, it is evident 1.6 is very much nearer the correct value of the root than 1.8.

270. If the given number is a perfect square and contains not more than ten significant figures, the exact root will be obtained in all cases. That the number is a perfect square may be suspected by the fact that there are one or more 9's or 0's following the fifth figure of the number expressing the third approximation, and that when the third approximation is expressed correct to five figures, the square of its last digit (or the second figure of this square when the square contains more than one figure) will be the same as the last digit of the given number. This will be illustrated by two examples.

EXAMPLE.— $\sqrt{3,749,602,756} = ?$

SOLUTION.—Pointing off, we obtain 37'49'60'27'56. The first two factors are evidently 6 and $37 \div 6 = 6.16+$, and the first approximation $\frac{6 + 6.16}{2} = 6.08$, or 6.1 to two figures.

$3749 \div 61 = 61.45+$; $\frac{61 + 61.45}{2} = 61.23-$, or 61.2 to three figures.

$$374960 \div 612 = 612.67973+; \quad \frac{612 + 612.67973}{2} = 612.33986+,$$

or 612.34 to five figures. But $4^2 = 16$, and as the last figure of the given number is also 6, and as the sixth and seventh figures are 9 and 8, respectively, we suspect that the given number is a perfect power. It may not be, however, for the reason that the figures 5, 7, and 2, preceding 6, may be different from the ones given without changing the value of the root to five figures. Hence, the only way to ascertain the fact beyond possibility of doubt is to square the root; doing so, it is found that $61,234^2 = 3,749,602,756$, which is therefore a perfect square.

Had all the figures of the given number been used in finding the third approximation, the result would have been as follows: $3,749,602,756 \div 612 = 612.6801+$, and $\frac{612 + 612.6801}{2} = 612.34005+$, or 612.34 to five figures, as before, or 61,234 after locating the decimal point. Ans.

271. If the given number contains not more than three periods of significant figures—that is, if it contains not more than five or six significant figures—and is a perfect power, the fact will be revealed when finding the second factor in

the third approximation, for the two factors will then be exactly equal.

EXAMPLE.— $\sqrt{.00095481} = ?$

SOLUTION.— $.00095481 = .00'09'54'81$ when pointed off into periods of two figures each. The first two significant figures are 9.5. The first factor is evidently 3 and the second factor $9.5 \div 3 = 3.16+$. The first approximation is $\frac{3 + 3.16}{2} = 3.08$, or 3.1 to two figures.

$954 \div 31 = 30.77+$; the second approximation is $\frac{31 + 30.77}{2} = 30.88+$ $= 30.9$ to three figures.

$95481 \div 309 = 309$; hence, .00095481 is a perfect power and the significant figures of the root are 309. There being one full cipher period following the decimal point, the root is .0309. Ans.

272. One more example will be given to show the student how to arrange his work when solving examples in square root by this method.

EXAMPLE.— $\sqrt{3,265.47} = ?$

SOLUTION.— $3,265.47 = 32'65.47.$

$33 \div 5 = 6.6$; $33 \div 6 = 5.5$; $6.6 - 5 = 1.6$; $6 - 5.5 = .5$; hence, use 6 for first factor.

$$\frac{6 + 5.5}{2} = 5.75, \text{ or } 5.8.$$

$$3,265 \div 58 = 56.29; \quad \frac{58 + 56.29}{2} = 57.14, \text{ or } 57.1.$$

$326,547 \div 571 = 571.886$; $\frac{571 + 571.886}{2} = 571.443$, or 571.44 to five figures. Therefore, $\sqrt{3,265.47} = 57.144$. Ans.

EXAMPLES FOR PRACTICE.

Find the square root of:

		Ans.	
(*a*)	186,624.	(*a*)	432.
(*b*)	2,050,624.	(*b*)	1,432.
(*c*)	29,855,296.	(*c*)	5,464.
(*d*)	.0116964.	(*d*)	.10815—.
(*e*)	198.1369.	(*e*)	14.076+.
(*f*)	994,009.	(*f*)	997.
(*g*)	2.375.	(*g*)	1.5411+.
(*h*)	1.625.	(*h*)	1.2748—.
(*i*)	.3025.	(*i*)	.55.
(*j*)	.571428.	(*j*)	.75593—.
(*k*)	.78125.	(*k*)	.88388+.

CUBE ROOT.

273. Cube root may be extracted in a manner similar to that just described for square root, the only essential differences being that the given number must be pointed off into periods of *three* figures each; the first period, if integral, may contain one, two, or three figures; and the number must be divided into three equal factors.

274. As might be expected, cube root is a longer operation than square root, but the method is similar and is no more difficult to remember or apply. As in the case of square root, it is unnecessary to use more than six significant figures in order to obtain five significant figures of the root. The method is best illustrated by an example. The student is advised to make a little table, containing the cubes of numbers from 1 to 9, similar to that here given.

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$

EXAMPLE.— $\sqrt[3]{389,247} = ?$

SOLUTION.— $389,247 = 389'247$ when pointed off into periods of three figures each. As in the case of square root, consider the first period only when finding the first approximation. In other words, divide 389 into three factors as nearly equal as possible. It is readily seen that 389 lies between $7^3 = 343$ and $8^3 = 512$; hence, the first figure of the root is 7. Now, assume that two of the equal factors are each equal to 7 and divide 389 by their product to obtain the third factor; that is, divide 389 by $7^2 = 49$. The result is $389 \div 49 = 7.93+$. Hence, $7 \times 7 \times 7.93+ = 389$, nearly. The average of these factors is $\frac{7+7+7.93}{3} = \frac{2 \times 7 + 7.93}{3} = 7.31$, or 7.3 to two figures, the first approximation.

Assuming 73 to be the value of two of the three equal factors, divide the first two periods of the given number by their product 73×73; that is, by 73^2, or 5,329. The result is $389,247 \div 5,329 = 73.04+$; that is, $7.3 \times 7.3 \times 73.04 = 389,247$, nearly. The average of the three factors is $\frac{2 \times 73 + 73.04}{3} = 73.01$, or 73.0 to three figures, the second approximation.

Assuming 730 to be two of the three equal factors, divide 389,247 by 730^2, or by 73^2, since the cipher at the right is not a significant figure and will not affect the result, obtaining for the third factor 73.0431.

The average of these three factors is $\frac{2 \times 73 + 73.0431}{3} = 73.0143$, or 73.014 to five figures. Ans.

NOTE.—The decimal point is located by applying the principle of Art. **267;** viz., there must be as many figures in the integral part of the root as there are periods in the integral part of the given number.

275. An inspection of the foregoing example shows that about the only respect in which the work of extracting cube root exceeds the work of extracting square root consists in squaring one number of two figures and one number of three figures. The work of division in finding the third factor is a little harder on account of the divisors being a little larger than when finding the second factor in square root.

276. If the given number contains an integral part, it is better to locate the decimal point as soon as possible, in order to prevent confusion, instead of waiting until the third approximation has been found.

EXAMPLE.— $\sqrt[3]{3.274} = ?$

SOLUTION.—The first period contains but one figure; therefore, we operate on three figures in order to have the first approximation correct to two figures (see **c,** Art. **261**). If 1 be chosen for one of the two equal factors, the third factor will be $3.27 \div 1^2 = 3.27$, and the difference between one of the equal factors and the third factor will be $3.27 - 1 = 2.27$. If 2 be chosen for one of the equal factors, the third one will be $3.27 \div 2^2 = .817+$, and the difference between this and one of the equal factors is $2 - .817 = 1.183$. Since 1.18 is less than 2.27, use 2 for one of the two equal factors. The first approximation is $\frac{2 \times 2 + .817}{3} = 1.60+$, or 1.6 to two figures. Since there is but one period in the integral part of the given number, the root is equal to 1 plus an interminable decimal, as the given number is not a perfect cube. Therefore, retain the decimal point in its present position through all the subsequent operations.

Assuming 1.6 to be one of the two equal factors, the third factor is $3.274 \div 1.6^2 = 3.274 \div 2.56 = 1.278+$, and the second approximation is $\frac{2 \times 1.6 + 1.278}{3} = 1.492+$, or 1.49 to three figures.

Assuming 1.49 to be one of the two equal factors, the other factor is $3.274 \div 1.49^2 = 3.274 \div 2.2201 = 1.474708+$; hence, the third approximation is $\frac{2 \times 1.49 + 1.474708}{3} = 1.484902+$, or 1.4849 to five figures. Ans.

The exact root to seven figures is 1.484886—.

277. The only case in cube root that will give any trouble in determining the fifth significant figure correctly is when the difference between the numbers representing the first and second approximations, expressed to two figures, is greater than one unit in the second figure. In the last example, the first approximation was 1.6 and the second 1.49, or 1.5 to two figures; the difference is .1, or one unit in the second figure. For numbers the significant part of whose first period is 2, the difference between the first and second approximations may differ by more than one unit in the second figure; in such cases, recalculate the second approximation, using for one of the equal factors the value of the second approximation to two figures as first determined. An example will illustrate this.

EXAMPLE.— $\sqrt[3]{.0027} = ?$

SOLUTION.— $.0027 = .002'700$ when pointed off into periods. But, the significant figures in the cube root of 2.7 will be the same as in the cube root of $.002'700$; therefore, find the cube root of 2.7 and locate the decimal point after the operation is finished.

If 1 be chosen as one of the equal factors, the third factor will be $2.7 \div 1^2 = 2.7$, and the first approximation is $\frac{2 \times 1 + 2.7}{3} = 1.56+$, or 1.6 to two figures. If 2 be chosen for one of the equal factors, the third factor is $2.7 \div 2^2 = .675$, and the first approximation is $\frac{2 \times 2 + .675}{3} = 1.55+$, or 1.6 to two figures.

Using 1.6 for one of the equal factors, the third factor is $2.7 \div 1.6^2 = 2.7 \div 2.56 = 1.054+$, and the second approximation is $\frac{2 \times 1.6 + 1.054}{3} = 1.418$, or 1.42 to three figures, or 1.4 to two figures. The difference between the first and second approximations is $1.6 - 1.4 = .2$, or two units in the second figure. Therefore, recalculate the second approximation, using 1.4 for one of the equal factors. The third factor is then equal to $2.7 \div 1.4^2 = 2.7 \div 1.96 = 1.377$, and the second approximation is $\frac{2 \times 1.4 + 1.377}{3} = 1.392+$.

Using 1.39 for one of the equal factors, the third factor is $2.7 \div 1.39^2 = 2.7 \div 1.9321 = 1.39744+$, and the third approximation is $\frac{2 \times 1.39 + 1.39744}{3} = 1.39248$, or 1.3925 to five figures. The root correct to nine figures is 1.39247665. Since the given number is wholly decimal and has no period composed entirely of ciphers, $\sqrt[3]{.0027} = .13925$. Ans.

Had 1.42 been used for one of the equal factors, the third approximation would have been 1.3930.

278. The remarks made in Art. **270** regarding the square root of perfect squares apply, with slight modifications, to the cube root of perfect cubes. If the given number is a perfect cube and contains not more than five periods, i. e., not more than $5 \times 3 = 15$ significant figures, the exact root can always be found. That the given number is a perfect cube will be suspected from the fact that the root ends in a string of 9's or 0's; that in one of the approximations the three factors become exactly equal; and that the last digit in the cube of the last figure of the root is the same as the last digit of the given number. An example will illustrate this.

EXAMPLE.— $\sqrt[3]{106,294,343.553} = ?$

SOLUTION.—The number when pointed off becomes $106'294'343.553$; hence, there are three figures in the integral part of the root. The first period 106 lies between $4^3 = 64$ and $5^3 = 125$. Trying 4 for one of the equal factors, the third factor is $106 \div 4^2 = 6.62+$. Trying 5, the third factor is $106 \div 5^2 = 4.24$; hence, use 5, and obtain for the first approximation $\frac{2 \times 5 + 4.24}{3} = 4.74+$. Using two periods and 47 for one of the equal factors, the third factor is $106,294 \div 47^2 = 106,294 \div 2,209 = 48.11+$, and the second approximation is $\frac{2 \times 47 + 48.11}{3} = 47.37$.

To find the third approximation, two, or three, or all four periods may be used, since the first two periods contain six significant figures, and hence will give the root correct to five figures (see **c,** Art. **261**). Using the first three periods, to avoid the decimal point, and 474 for one of the equal factors, the third factor is $106,294,343 \div 474^2 = 106,294,343 \div 224,676 = 473.1005+$, and the third approximation is $\frac{2 \times 474 + 473.1005}{3} = 473.7001+$, or 473.70 to five figures. It will be noticed that the results obtained for the second and third approximations are alike and the last digit in $7^3 = 343$ is the same as the last significant figure of the given number; hence, it is at once suspected that the given number is a perfect power, and this is proved by cubing the root. Therefore, $\sqrt[3]{106,294,343.553} = 473.7$. Ans.

279. Square and cube root are two of the most important operations described in Arithmetic, and the student is earnestly advised to thoroughly familiarize himself with the process. Few practical problems involving mensuration arise that do not require the extraction of the square

or cube root. For instance, to find the diameter of a circle that will contain a given area requires the extraction of square root; to find the diameter of a sphere that will contain a given volume requires the extraction of cube root.

EXAMPLES FOR PRACTICE.

Find the cube root of:

(*a*) 78,347.809639.
(*b*) 2.
(*c*) 4,180,769,192.462.
(*d*) .696.
(*e*) .375.
(*f*) 513,229.783302144.

Ans. (*a*) 42.79. (*b*) 1.2599+. (*c*) 1,611.0—. (*d*) .88621—. (*e*) .72112+. (*f*) 80.064.

TABLE METHOD OF EXTRACTING SQUARE AND CUBE ROOT.

280. By means of the table of Squares, Cubes, Fourth, and Fifth Powers, which contains the squares and cubes of numbers from 1 to 10, varying by tenths, and the first five figures of the fourth and fifth powers of the same numbers, the first three, and frequently the first four, significant figures of the square root or cube root of any number can be readily determined. The remaining figures can then be easily determined in the same manner as the third approximation in the preceding pages.

The student is advised to use the table in all cases, as it will greatly shorten his work.

281. By the aid of this table the first two significant figures of the root can be obtained directly and one more by a slight calculation. For example, suppose it is desired to find the first three significant figures of $\sqrt{5,269.73}$. Pointing off into periods and moving the decimal point so that it falls between the first and second periods, the number becomes 52.69′73; in other words, the significant figures of $\sqrt{5,269.73}$ are the same as for $\sqrt{52.6973}$. Since four figures only are given in the table, reduce the given number to four figures. The problem then becomes: find the first three figures of $\sqrt{52.70}$. Referring to the table, 52.70 lies between

SQUARES, CUBES, FOURTH, AND FIFTH POWERS.

No.	Square.	Cube.	4th Power.	5th Power.	No.	Square.	Cube.	4th Power.	5th Power.
1.0	1.00	1.000	1.0000	1.0000	5.5	30.25	166.375	915.06	5,032.8
1.1	1.21	1.331	1.4641	1.6105	5.6	31.36	175.616	983.45	5,507.3
1.2	1.44	1.728	2.0736	2.4883	5.7	32.49	185.193	1,055.6	6,016.9
1.3	1.69	2.197	2.8561	3.7129	5.8	33.64	195.112	1,131.6	6,563.6
1.4	1.96	2.744	3.8416	5.3782	5.9	34.81	205.379	1,211.7	7,149.2
1.5	2.25	3.375	5.0625	7.5938	6.0	36.00	216.000	1,296.0	7,776.0
1.6	2.56	4.096	6.5536	10.485	6.1	37.21	226.981	1,384.6	8,446.0
1.7	2.89	4.913	8.3521	14.199	6.2	38.44	238.328	1,477.6	9,161.3
1.8	3.24	5.832	10.498	18.896	6.3	39.69	250.047	1,575.3	9,924.4
1.9	3.61	6.859	13.032	24.761	6.4	40.96	262.144	1,677.7	10,737
2.0	4.00	8.000	16.000	32.000	6.5	42 25	274.625	1,785.1	11,603
2.1	4.41	9.261	19.448	40.841	6.6	43.56	287.496	1,897.5	12,523
2.2	4.84	10.643	23.426	51.536	6.7	44.89	300.763	2,015.1	13,501
2.3	5.29	12.167	27.984	64.363	6.8	46.24	314.432	2,138.1	14,539
2.4	5.76	13.824	33.178	79.626	6.9	47.61	328.509	2,266.7	15,640
2.5	6.25	15 625	39.063	97.656	7.0	49 00	343.000	2,401.0	16,807
2.6	6.76	17.576	45.698	118.81	7.1	50.41	357.911	2,541.2	18,042
2.7	7.29	19.683	53.144	143.49	7.2	51.84	373.248	2,687.4	19,349
2.8	7.84	21.952	61.466	172.10	7.3	53.29	389.017	2,839.8	20,731
2.9	8.41	24.389	70.728	205.11	7.4	54.76	405.224	2,998.7	22,190
3.0	9.00	27.000	81.000	243.00	7.5	56.25	421.875	3,164.1	23.730
3.1	9 61	29.791	92.352	286.29	7.6	57.76	438.976	3,336.2	25,355
3.2	10.24	32.768	104.86	335.54	7.7	59.29	456.533	3,515.3	27,068
3.3	10.89	35.937	118.59	391.35	7.8	60.84	474.552	3,701.5	28,872
3.4	11.56	39.304	133.63	454.35	7.9	62.41	493.039	3,895.0	30,771
3.5	12.25	42.875	150.06	525.22	8.0	64.00	512.000	4,096.0	32,768
3.6	12.96	46.656	167.96	604.66	8.1	65.61	531.441	4,304.7	34,868
3.7	13.69	50.653	187.42	693.44	8.2	67.24	551.368	4,521.2	37,074
3.8	14.44	54.872	208.51	792.35	8.3	68.89	571.787	4,745.8	39,390
3.9	15.21	59.319	231.34	902.24	8.4	70.56	592 704	4,978.7	41,821
4.0	16.00	64.000	256.00	1,024.0	8.5	72.25	614.125	5,220.1	44,371
4.1	16.81	68.921	282.58	1,158.6	8.6	73.96	636.056	5,470.1	47,043
4.2	17.64	74.088	311.17	1,306.9	8.7	75.69	658.503	5,729.0	49,842
4.3	18.49	79.507	341.88	1,470.1	8.8	77 44	681.472	5,997.0	52,773
4.4	19.36	85.184	374.81	1,649.2	8.9	79.21	704.969	6,274.2	55,841
4.5	20.25	91.125	410.06	1,845.3	9.0	81.00	729.000	6,561.0	59,049
4.6	21.16	97.336	447.75	2,059.6	9.1	82.81	753.571	6,857.5	62,403
4.7	22.09	103.823	487.97	2,293.5	9.2	84.64	778.688	7,163.9	65,908
4.8	23.04	110.592	530.84	2,548.0	9.3	86.49	804.357	7,480.5	69,569
4.9	24.01	117.649	576.48	2,824.8	9.4	88.36	830.584	7,807.5	73,390
5.0	25.00	125.000	625.00	3,125.0	9.5	90.25	857.375	8,145.1	77,378
5.1	26.01	132.651	676.52	3,450.3	9.6	92.16	884.736	8,493.5	81,537
5.2	27.04	140.608	731.16	3,802.0	9.7	94.09	912.673	8,852.9	85,873
5.3	28.09	148.877	789.05	4,182.0	9.8	96.04	941.192	9,223.7	90,392
5.4	29.16	157.464	850.31	4,591.7	9.9	98.01	970.299	9,606.0	95,099

$51.84 = 7.2^2$ and $53.29 = 7.3^2$; hence, the first two figures of the root are 7.2. Find the difference between the two numbers in the table between which the given number falls and call it the **first difference;** thus, $53.29 - 51.84 = 1.45 =$ the first difference. Find the difference between the lower number in the table and the given number and call it the **second difference;** thus, $52.70 - 51.84 = .86 =$ the second difference. Divide the second difference by the first difference, and the first figure of the quotient, if the quotient is .05 or greater, will be the third figure of the root, when reduced to one figure. If the quotient is less than .05, the third figure of the root is a cipher. Thus, $.86 \div 1.45 = .59+$, or .6 when reduced to one figure. Therefore, the first three figures of $\sqrt{52.70}$ are 7.26. Since the integral part of the given number contains two periods, there are two figures in the integral part of the root; therefore, $\sqrt{5{,}269.73} = 72.6$ to three figures. Ans.

282. The cube root is found to three significant figures in exactly the same way, as shown in the following example:

EXAMPLE.—Find the first three figures of $\sqrt[3]{.0625}$.

SOLUTION.—Pointing off and placing the decimal point between the first and second significant periods, the result is 62.500. Referring to the table, the first two figures of the root are 3.9; the first difference is $64.000 - 59.319 = 4.681$; the second difference is $62.500 - 59.319 = 3.181$; $3.181 \div 4.681 = .67+$, or .7 to one figure. Therefore, $\sqrt[3]{62.5} = 3.97$, and $\sqrt[3]{.0625} = .397$ to three significant figures. Ans.

283. Having found the first three significant figures by means of the table, find the fourth and fifth figures in the usual manner by using the first three figures in finding the third approximation.

For example, find the cube root of 126.57 to five figures. Referring to the table, the first two figures are 5.0. The first difference is $132.651 - 125.000 = 7.651$; the second difference is $126.57 - 125.000 = 1.57$; $1.57 \div 7.651 = .20+$. Hence, the first three figures are 5.02. Using 5.02 for one of the equal factors, the third factor is $126.57 \div 5.02^2$

$= 5.02253+$, and the third approximation is $\frac{2 \times 5.02 + 5.02253}{3}$ $= 5.02084+$, or $\sqrt[3]{126.57} = 5.0208$ to five figures. Ans.

284. If more than five significant figures of the square or cube root are desired, use the five figures of the third approximation for one of the equal factors and calculate the unknown factor to as many figures as are desired plus one; the next approximation will be correct to at least nine figures, if the unknown factor has been calculated to ten figures.

ROOTS OF FRACTIONS.

285. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator and the root of the denominator for a new denominator.

EXAMPLE 1.—What is the square root of $\frac{9}{64}$?

SOLUTION.— $\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$. Ans.

EXAMPLE 2.—What is the square root of $\frac{5}{8}$?

SOLUTION.— $\sqrt{\frac{5}{8}} = \sqrt{.625} = .79057-$, since $\frac{5}{8} = .625$. Ans.

EXAMPLE 3.—What is the cube root of $\frac{27}{64}$?

SOLUTION.— $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$. Ans.

EXAMPLE 4.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996+$. Ans.

286. Rule.—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

(*a*) $\sqrt{\frac{9}{16}} = ?$
(*b*) $\sqrt[3]{\frac{27}{1728}} = ?$
(*c*) $\sqrt[3]{\frac{27}{375}} = ?$
(*d*) $\sqrt[3]{\frac{560}{128}} = ?$

Ans. (*a*) $\frac{3}{4}$. (*b*) $\frac{1}{4}$. (*c*) .41602—. (*d*) 1.6355+.

FOURTH ROOT.

287. The fourth root may be found by a method similar to that just described for extracting square and cube roots, dividing the given number into periods of four figures each and resolving the given number into four equal factors. It is generally easier and shorter, however, to extract the square root and then extract the square root of the result. For example, to extract the fourth root of 5,735,796,283.8016, which is a perfect fourth power (and consequently a perfect square, also), the square root would be extracted in the usual manner, obtaining 75,735.04. The square root of this result would then be extracted, obtaining 275.2. In other words $\sqrt[4]{5,735,796,283.8016} = \sqrt{\sqrt{5,735,796,283.8016}} = \sqrt{75,735.04} = 275.2$.

The fourth root is very seldom required and can always be found as just described.

FIFTH ROOT.

288. The fifth root is required oftener than the fourth root, but nevertheless it is seldom necessary to extract it. The method is the same in principle as that explained for cube root. The given number is divided into periods of five figures each and resolved into five equal factors; the first period may contain one, two, three, four, or five figures. As in the case of cube root, it is advisable to construct a little table giving the fifth powers of the nine digits, similar to that here given. An example will illustrate the process.

$1^5 =$	1
$2^5 =$	32
$3^5 =$	243
$4^5 =$	1,024
$5^5 =$	3,125
$6^5 =$	7,776
$7^5 =$	16,807
$8^5 =$	32,768
$9^5 =$	59,049

EXAMPLE.— $\sqrt[5]{5,186.42} = ?$

SOLUTION. — The first period 5186 lies between $5^5 = 3125$ and $6^5 = 7776$; hence, the root is 5 plus an interminable decimal. Trying 5 as one of the four equal factors and dividing the first period by their product to find the fifth factor, we have $\frac{5186}{5 \times 5 \times 5 \times 5} = 5186 \div 5^4 = 5186 \div 625 = 8.29+$. Trying 6 as one of the four equal factors, the fifth factor is $5186 \div 6^4 = 5186 \div 1296 = 4.00+$. Since the difference between 6 and 4 is less than the difference between 5 and 8.29, use 6 as one of the equal factors. Then, $5186 = 6 \times 6 \times 6 \times 6 \times 4$. The average of these factors is $\frac{6+6+6+6+4}{5} = \frac{4 \times 6 + 4}{5} = \frac{28}{5} = 5.6$, the first approximation.

Using 5.6 as one of the equal factors, the fifth factor is $5186.42 \div 5.6^4 = 5186 \div 983.4496 = 5186 \div 983.4000$ (using but four significant figures, since only three figures of the second approximation are required—see **b**, Art. **261**) $= 5.273$, and the second approximation is $\frac{4 \times 5.6 + 5.273}{5} = 5.534+$, or 5.53 to three figures.

Using 5.53 as one of the equal factors, the fifth factor is $5186.42 \div 5.53^4 = 5186.42 \div 935.191 = 5.54584+$, and the third approximation is 5.53316+, or 5.5332 to five figures. Ans.

The exact root to seven figures is 5.533164.

289. In order that the fifth significant figure of the fifth root of a number may be correct, it is absolutely essential that the third significant figure of the second approximation be correct. In the last example, it will be noticed that the difference between the first and second approximations is $5.60 - 5.53 = .07$, which is less than one unit in the second figure, but very near to it. Had this difference been as much as 1, or had it exceeded one unit in the second figure, it would have been advisable to recalculate the second approximation.

290. The labor involved in extracting the fifth root is very much greater than that necessary to extract the cube root, chiefly on account of raising numbers to the fourth power. This labor may be shortened considerably in the following manner:

Consider any number, as 4. Now, $4^4 = 4 \times 4 \times 4 \times 4 = (4 \times 4) \times (4 \times 4) = 16 \times 16 = 256$. In other words, to

raise a number to the fourth power, square the number and then square the square. Now, the square of any number contains twice as many significant figures as the number or twice as many less 1; the cube of any number contains three times as many significant figures as the number or three times as many less one or two; the fourth power will contain four times as many or four times as many less one, two, or three; and so on. Hence, the fourth power of a number containing two figures will contain five, six, seven, or eight figures; and of one containing three figures, nine, ten, eleven, or twelve figures. In determining the fifth factor for the second approximation, only four figures of the fourth power are required, and in determining the fifth factor for the third approximation, only six figures of the fourth power are required. Therefore, any method that will enable us to dispense with unnecessary figures will lessen the work. The following method, which will also apply to any case of multiplication when only a certain definite number of figures are desired in the product, is the best we know of; it is best illustrated by an example.

EXAMPLE.—Multiply 467,295 by 634,137 and obtain six figures of the product correct.

SOLUTION.—

(*a*)		(*b*)
467295		4'6'7'2'9'5
634137		634137
2803770		2803770
140188	5	140188
18691	80	18691
467	295	467
140	1885	140
32	71065	32
2963290	49415	2963288, or 296329 to six figures.

EXPLANATION.—Instead of beginning with the last, or right-hand, digit, as in ordinary multiplication, begin with the first, or left-hand, digit of the multiplier and multiply in the ordinary manner, the product being 2,803,770. Multiply by the second digit and write the first figure obtained

in the second partial product one place to the right of the last figure of the first partial product. So proceed until all the partial products have been found, adding them to find the entire product. The result is shown at (*a*). Now, in order to have six figures of the product correct, seven figures should be obtained (see **a**, Art. **261**). It is therefore evident that all figures to the right of the vertical line in (*a*) are unnecessary. Hence, proceed as shown in (*b*). The first partial product contains seven figures—all that are required; therefore, cut off the figure 5 in the multiplicand when finding the second partial product, but multiply it by 3 in order to determine how much to carry. Thus, say mentally "three times five is fifteen," and carry 1. Then say "three times nine is twenty-seven and one is twenty-eight," etc. When multiplying by the next digit 4, cut off the second figure from right of the multiplicand, but carry the 3 that is obtained by multiplying 9 by 4, and say "four times two is eight and three is eleven." Proceeding in this manner, no figure of any of the partial products will extend beyond the place occupied by the seventh figure of the entire product.

291. The operation of division may be shortened in a similar manner to that just described for multiplication. Perform the division in the usual manner until the number of significant figures in the quotient equals the number obtained by subtracting the number of significant figures in the divisor from the number desired in the quotient plus three; then cut off one figure from the right of the divisor before finding the next figure of the quotient; cut off the second figure from the right of the divisor before finding the succeeding figure of the quotient; and so on until the quotient contains one more than the required number of figures. It is here assumed that the dividend and divisor do not contain more than one significant figure in excess of the number required in the quotient. (See **a** and **b**, Art. **261.**)

EXAMPLE.—Divide 71,346.247 by 27,846.392 and obtain five significant figures of the quotient correct.

SOLUTION.—

```
71346.20 | 27'8'4'6'.4
556928     2.56213, or 2.5621 to five figures.  Ans.
1565340
1392320
 173020
 167078
   5942
   5569
    373
    278
     95
     83
     12
```

EXPLANATION.—Since five significant figures are required in the quotient, the dividend and divisor are limited to six significant figures. The number of significant figures required in the quotient before beginning to cut off figures from the divisor is $5 + 3 - 6 = 8 - 6 = 2$; hence, before finding the third figure of the quotient, cut off the figure 4 from the right of the divisor, but multiply 4 by 6 in order to see how much to carry. Thus, say "six times four is twenty-four," and carry 2; then, say "six times six is thirty-six and two is thirty-eight," and write 8 and carry 3; and so on. Before finding the fourth figure of the quotient, cut off the next figure 6 of the divisor. The student will find it convenient to place the divisor at the right of the dividend with the quotient underneath, as shown. This arrangement saves space and brings each figure of the quotient directly under the divisor, making the multiplication easier.

292. To locate the decimal point in the quotient, the easiest way is to proceed as follows: Move the decimal point in the divisor to the right until it follows the right-hand figure; that is, make the divisor a whole number; move the decimal point in the dividend as many decimal places to the right as it was moved in the divisor, annexing ciphers if necessary. If the dividend will contain the divisor one or more times, there will be as many figures in the integral part of the quotient as there are figures left in the dividend after finding the first remainder plus one. If the dividend will not contain the divisor, annex ciphers to follow the decimal point until the dividend contains the divisor, and the first significant figure of the quotient will then be located as many decimal places to the right of the decimal point as there were ciphers annexed. For instance, $.046 \div 21.76 = 4.6 \div 2176. = 4.600 \div 2{,}176. = .002+$; $4.6 \div 21.76 = 460. \div 2{,}176. = 460.0 \div 2{,}176 = .2+$; $460 \div 21.76 = 46{,}000. \div 2{,}176 = 21.+$. In the last example, $71{,}346.2 \div 27{,}846.4 = 713{,}462. \div 278{,}464. = 2+$.

293. Having shown how the work of calculation may be greatly reduced, an example is given on the following page, showing all the figures used in extracting the fifth root, each operation being numbered in the order in which it is performed.

294. When the sixth significant figure of the third approximation is 5, it is not always advisable to increase the fifth figure by one. To ascertain whether or not the fifth figure should be increased, recalculate the third approximation, using for one of the equal factors the third approximation first found, correct to *four* figures; if the sixth significant figure is then 5+, increase the fifth figure by 1.

EXAMPLE 1.— $\sqrt[5]{3.056} = ?$

SOLUTION.—Using the first two significant figures and trying 1 for the equal factors, the fifth factor is $3.1 \div 1^4 = 3.1$, and the first approximation is $\frac{4 \times 1 + 3.1}{5} = 1.42$. Since $3.1 - 1 = 2.1$ is less than 2.5 (see Remark in example referred to in Art. **293**), it is not necessary to try 2 for one of the equal factors.

$$3.056 \div 1.4^4 = .795+. \quad \frac{4 \times 1.4 + .795}{5} = 1.279+.$$

Since the difference between 1.42, the first approximation, and 1.279 the second approximation, is greater than one unit in the second figure, try 1.3 for one of the equal factors and recalculate the second approximation.

$$3.056 \div 1.3^4 = 1.069+. \quad \frac{4 \times 1.3 + 1.069}{5} = 1.253+.$$

EXAMPLE.— $\sqrt[5]{8,269} = ?$

SOLUTION.—

(1)

6
6
36
36
108
.216
1296

(2)

8269 | 1296
7776 6.38+
493
388
105
103
2

(3)

$$\frac{4 \times 6 + 6.38}{5} = 6.07+$$

(6)

$$\frac{4 \times 6.1 + 5.974}{5} = \frac{30.374}{5} = 6.074+$$

(4)

6.1
6.1
366
61
37.21
37.21
11163
2604
74
3
13844

(5)

8269.0 | 1384
6920 5.974
13490
12456
1034
968
66
55
11

(7)

6.07
6.07
3642
4249
36.8449
36.'8'4'5
36.845
110535
22 1070
2 9476
1473
184
1357.553

(8)

8269.00 | 13'57.'5'5
814530 6.09 112
123700
122179
1521
1357
164
135
29
27
2

(9)

$$\frac{4 \times 6.07 + 6.09112}{5} =$$

$$\frac{30.37112}{5} = 6.07422$$

= 6.0742 to five figures. Ans.

REMARK.—It was unnecessary to try 7 for one of the equal factors, because there was but a very little difference between the fifth factor 6.38 and one of the equal factors 6; in fact, when this difference is not greater than 2.5 units, it is unnecessary to try the next higher number for one of the equal factors. In this case the difference is 6.38 − 6 = .38, or less than 1 unit.

Since the difference, 1.3 − 1.253 = .047, is less than one unit in the second figure, use 1.25 for one of the equal factors in finding the third approximation.

$3.056 \div 1.25^4 = 1.25173+$. $\frac{4 \times 1.25 + 1.25173}{5} = 1.25034+$,

or 1.2503 to five figures. Ans.

The exact root to seven figures is 1.250347.

EXAMPLE 2.— $\sqrt[5]{3} = ?$

SOLUTION.—Trying 1.3 for one of the equal factors (see last example), $3 \div 1.3^4 = 1.050+$. $\frac{4 \times 1.3 + 1.050}{5} = 1.25$.

Using 1.25 for one of the equal factors, the fifth factor is $3 \div 1.25^4 = 1.22879+$, and the third approximation is $\frac{4 \times 1.25 + 1.22879}{5} = 1.24575+$. Since the sixth figure is 5, it will be well to recalculate the third approximation, using 1.246 for one of the equal factors. Hence, $3 \div 1.246^4 = 1.244656+$, and the third approximation is

$\frac{4 \times 1.246 + 1.244656}{5} = 1.245731+$, or 1.2457 to five figures. Ans.

The exact root to seven figures is 1.245731.

295. The fifth root is very seldom required; the most prominent case in practice arises in connection with finding the diameter of a pipe that will discharge a required amount of water, the head and length of pipe being known. It is also required in connection with certain problems in mine ventilation. The fourth root is used even less frequently than the fifth root. Roots higher than the fifth are never required.

TABLE METHOD OF EXTRACTING THE FIFTH ROOT.

296. In exactly the same way as in the case of square and cube roots, the first three significant figures of the fifth root may be found by means of the table of the powers of numbers.

EXAMPLE.— $\sqrt[5]{238.75} = ?$

SOLUTION.—Referring to the table, the first two figures are 2.9; the first difference is 243.00 − 205.11 = 37.89; the second difference is 238.75 − 205.11 = 33.64; 33.64 ÷ 37.89 = .88+, or .9 to one figure. Therefore, $\sqrt[5]{238.75} = 2.99$ to three figures.

297. When finding the fifth root of numbers whose first period contains but one significant figure, carry the quotient obtained by dividing the second difference by the first difference to three decimal places, and if the third figure is 5 or a greater digit, increase the second figure by 1 and add these two figures of the quotient to those previously found for the third and fourth figures of the root. Then use all four figures when finding the third approximation.

EXAMPLE.— $\sqrt[5]{3} = ?$

SOLUTION.—Referring to the table, the first two figures are 1.2; the first difference is $3.7129 - 2.4883 = 1.2246$; the second difference is $3 - 2.4883 = .5117$; $.5117 \div 1.2246 = .417+$, or .42 to two figures. Hence, assume that one of the equal factors is 1.242; the fifth factor is $3 \div 1.242^4 = 1.26076$, and the third approximation is $\frac{4 \times 1.242 + 1.26076}{5}$ $= 1.245752$. Since the sixth figure is 5, recalculate the third approximation, using the result just found correct to four figures for one of the equal factors (see Example 2, Art. **294**). The result is 1.2457. Ans.

ARITHMETIC.

(SECTION 6.)

INTRODUCTION.

The subject of ratio and proportion is one of the most useful of all the subjects that are taught in Arithmetic. The student will find frequent use for the principles treated of in the following pages, and is requested to study them with great care.

The student should carefully study the definitions, constantly referring to them from time to time as he progresses with the subject; he should note particularly those articles relating to inverse ratio and inverse proportion. The idea of inverse proportion is usually a difficult one for the student to grasp; but a careful study of Art. **330** and of the examples in Arts. **331** and **332** will make the matter clear to him.

Although some of the examples included between Arts. **311** and **334,** inclusive, may be solved by other methods than the use of proportion, all the examples included between the above articles, and those of similar nature included in the Question Paper, must be solved by applying the principles of proportion; no other method of solution will be accepted. The student should study very carefully Arts. **309, 310, 324,** and **325;** they are very important, and they should be thoroughly understood.

The subject of compound proportion as treated in ordinary textbooks on Arithmetic usually proves of considerable difficulty to the student. The method here given, while not entirely new, presents the matter in a clearer light, we believe, than any other we have ever seen.

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RATIO.

298. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$ or .2 of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

299. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$; 20 : 4, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4.* The ratio of 4 to 20 would be expressed thus: $4 \div 20$; 4 : 20, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third is rapidly growing in favor, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of Algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, 20 | 4.

300. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet;** when considered separately, the first term is called the **antecedent,** and the second term the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent, and 4 the consequent.

301. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal** ratio. The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i.e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

302. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

303. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20:4 is 5, it is the quotient obtained by dividing the antecedent by the consequent.

304. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied, or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

305. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio; and dividing the antecedent or multiplying the consequent divides the ratio.

306. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20:4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

307. What is the value of the ratio of

(*a*) 98 to 49?
(*b*) \$45 to \$9?
(*c*) $6\frac{1}{4}$ to $\frac{2}{4}$?
(*d*) 3.5 to 4.5?
(*e*) The inverse ratio of 76 to 19?
(*f*) The inverse ratio of 49 to 98?
(*g*) The inverse ratio of 18 to 24?
(*h*) The inverse ratio of 9 to 15?
(*i*) The ratio of 10 to 3, multiplied by 3?
(*j*) The ratio of 35 to 49, multiplied by 7?
(*k*) The ratio of 18 to 64, divided by 9?
(*l*) The ratio of 14 to 28, divided by 5?

Ans. (*a*) 2. (*b*) 5. (*c*) $12\frac{1}{2}$. (*d*) $.77\frac{7}{9}$. (*e*) $\frac{1}{4}$. (*f*) 2. (*g*) $1\frac{1}{3}$. (*h*) $1\frac{2}{3}$. (*i*) 10. (*j*) 5. (*k*) $\frac{1}{32}$. (*l*) $\frac{1}{10}$.

---

308. Instead of expressing the value of a ratio by a single number, as above, it is customary to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$ or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

309. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$, it may be done by simply dividing the exponents by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

310. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$ (this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), it follows that the antecedent and consequent may always be multiplied or divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$. Also, $4^2 = 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$.

The statement may be proved for roots in the same manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$ and $\sqrt[3]{18^3} = 18$; and, $24 : 18 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$ and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$.

NOTE.—If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations described in Art. **310** cannot be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form, it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

PROPORTION.

311. Proportion is an equality of ratios, the cquality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8:4 and 6:3, which both have the same value 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$
$$8 : 4 = 6 : 3 \qquad (2)$$
$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

312. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably entirely supersede the first form. In this paper we shall adopt the second form, unless some statement can be made clearer by using the third form.

313. A proportion may be *read* in two ways. The old way to read the above proportion was—*8 is to 4 as 6 is to 3;* the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

314. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8:4 is a couplet, and so is 6:3.

315. The numbers forming the proportion are called **terms;** and they are numbered consecutively from left to right, thus:

first second third fourth
$$8 : 4 = 6 : 3$$

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

316. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

317. A **direct proportion** is one in which both couplets are direct ratios.

318. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written $8 : 4 = 6 : 3$; i. e., the second ratio (couplet) must be inverted.

319. Proportion forms one of the most useful sections of Arithmetic. In our grandfathers' Arithmetics, it was called "The rule of three."

320. Rule I.—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

$$17 : 51 = 14 : 42.$$

$17 \times 42 = 51 \times 14$, since both products equal 714.

321. Rule II.—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion $17 : 51 = \quad : 42$?

SOLUTION.—Applying rule II, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

322. Rule III.—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion $\quad : 51 = 14 : 42$?

SOLUTION.—Applying rule III, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans.

323. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written,

$$x : 51 = 14 : 42$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

324. If the same (addition and subtraction excepted) operations be performed upon *all* of the terms of a proportion, the proportion is not thereby destroyed. In other words, if all of the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; if (4) the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the third form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to prove is that, if any of the five operations enumerated above be performed upon all of the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since both equal $\frac{1}{2}$.

325. If both terms of either couplet be multiplied or both divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **304.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying rules II or III. Thus, the proportion $45 : 9 = x : 7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5 : 1 = x : 7.1$, whence $x = 7.1 \times 5 \div 1 = 35.5$. (See note in Art. **310.**)

326. The principle of all calculations in proportion is this: *Three of the terms are always given, and the remaining one is to be found.*

327. EXAMPLE.—If 4 men can earn \$25 in one week, how much can 12 men earn in the same time?

SOLUTION.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is:

"What is it I want to find?"

In this case it is dollars. We have two sets of men, one set earning \$25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to \$25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

The amount 12 men earn : \$25 = 12 men : 4 men,

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x or the required term occupies, the problem could be stated as any of the following forms, the value of x being the same in each :

(*a*) \$25 : the amount 12 men earn = 4 men : 12 men ; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(*b*) 4 men : 12 men = \$25 : the amount 12 men earn ; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(*c*) 12 men : 4 men = the amount 12 men earn : \$25 ; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

328. If the proportion is an inverse one, first form it as though it were a direct proportion, and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

329. Find the value of x in each of the following:

		Ans.
(*a*)	\$16 : \$64 :: x : \$4.	(*a*) $x = \$1$.
(*b*)	x : 85 :: 10 : 17.	(*b*) $x = 50$.
(*c*)	24 : x :: 15 : 40.	(*c*) $x = 64$.
(*d*)	18 : 94 :: 2 : x.	(*d*) $x = 10\frac{4}{9}$.
(*e*)	\$75 : \$100 = x : 100.	(*e*) $x = 75$.
(*f*)	15 pwt. : x = 21 : 10.	(*f*) $x = 7\frac{1}{7}$ pwt.
(*g*)	x : 75 yd. = \$15 : \$5.	(*g*) $x = 225$ yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches.

INVERSE PROPORTION.

330. In Art. **318**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example ; in such cases the proportion can be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth ; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

331. EXAMPLE.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5}$ = 58.8 days.

EXAMPLES FOR PRACTICE.

332. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. The weight of any gas (the volume and pressure remaining the same) varies inversely as the absolute temperature. If a certain quantity of some gas weighs 2.927 lb. when the absolute temperature is 525°, what will the same volume of gas weigh when the absolute temperature is 600°, the pressure remaining the same? Ans. 2.561+ lb.

4. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 562°, what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases?

Ans. 407°, very nearly.

POWERS AND ROOTS IN PROPORTION.

333. It was stated in Art. **309** that a ratio could be raised to any power or any root of it might be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power or the same root of both terms must be taken.*

334. EXAMPLE.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

SOLUTION.—This is evidently a direct proportion. Hence, we write

$$6^3 : 8^3 = x : 180.$$

Dividing both terms of the first couplet by 2^3 (see Art. **310**),

$$3^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence, $x = \frac{27 \times 180}{64} = 75\frac{15}{16}$ pounds. Ans.

EXAMPLE.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\frac{15}{16}$ pounds?

SOLUTION.—Since the weights of any two spheres are to each other as the cubes of their diameters, we have the proportion

$$180 : 75\tfrac{15}{16} = 8^3 : x^3;$$

x, the required term, must be cubed, because the other term of the couplet is cubed (see Art. **333**). But, $8^3 = 512$; hence,

$$180 : 75\tfrac{15}{16} = 512 : x^3, \text{ or } x^3 = \frac{75\frac{15}{16} \times 512}{180} = 216;$$

whence, $x = \sqrt[3]{216} = 6$ inches. Ans.

335. Since taking the same root of all of the terms of a proportion does not change its value (Art. **324**), the above example might have been solved by extracting the cube root of all of the numbers, thus obtaining $\sqrt[3]{180} : \sqrt[3]{75\frac{15}{16}} = 8 : x$; whence,

$$x = \frac{8 \times \sqrt[3]{75\frac{15}{16}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\frac{75\frac{15}{16}}{180}} = 8\sqrt[3]{\frac{1,215}{2,880}} = 8\sqrt[3]{\frac{27}{64}} =$$

$8 \times \frac{3}{4} = 6$ inches. The process, however, is longer and is not so direct, and the first method is to be preferred.

336. If two cylinders have *equal* volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}.$$

Since neither 12 nor 15 are perfect squares, we square all of the terms (Arts. **335** and **324**) and obtain

$$81 : x^2 = 15 : 12; \text{ whence } x^2 = \frac{81 \times 12}{15} = 64.8,$$

and $x = \sqrt{64.8} = 8.05$ inches = diameter of 15-inch cylinder.

EXAMPLES FOR PRACTICE.

337. Solve the following examples:

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 30 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet? Ans. $2\frac{1}{4}$ times as bright.

2. In the last example, suppose that the object had been 40 feet from the gas jet; how bright would it have been compared with its brightness at 30 feet from the gas jet? Ans. $\frac{9}{16}$ as bright.

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the

source. If a man can just distinguish the time indicated by his watch, 50 feet from a certain light, at what distance could he distinguish the time from a light 3 times as powerful? Ans. 86.6+ feet.

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot?

Ans. 68,034 cu.ft. per min., nearly.

5. In the last example, suppose that 70,000 cubic feet per minute had been required; what would be the pressure necessary for this quantity? Ans. 3.81+ lb. per sq. ft.

CAUSES AND EFFECTS.

338. Many examples in proportion may be more easily solved by using the principle of *cause and effect*. That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called a **cause,** and the change or alteration, or thing accomplished, is the **effect.**

339. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the first cause is to the second cause as the first effect is to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause, and x pounds is the second effect. Hence, we may write

1st cause		*2d cause*		*1st effect*		*2d effect*
3	:	7	=	1,400	:	x,

whence $x = \frac{7 \times 1{,}400}{3} = 3{,}266\frac{2}{3}$ pounds.

340. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

341. All the cases of proportion so far considered have been cases of **simple proportion;** i. e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects or all four consist of more than two numbers. We will illustrate this by an

EXAMPLE.—If 40 men earn $1,280 in 16 days, how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn something, 16 days is also a cause. For the same reason, 36 men and 31 days are also causes. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. $1,280 is the first effect and $\$x$ is the second effect. Hence, we write

1st cause		*2d cause*		*1st effect*		*2d effect*
40 16	:	36 31	=	1,280	:	x

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. **299**), and the above becomes

$$\begin{matrix}40\\16\end{matrix} \Big| \begin{matrix}36\\31\end{matrix} = 1{,}280 \Big| x.$$

In the last expression, the product of all of the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or } x = \frac{36 \times 31 \times \overset{2}{\cancel{1280}}}{\underset{\cancel{40}}{\cancel{40}} \times \underset{\cancel{16}}{\cancel{16}}} = \$2{,}232. \quad \text{Ans.}$$

342. The above might have been solved by canceling factors of the numbers in the original proportion. For if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus, 16 is contained in

$$\begin{matrix}\cancel{40}\\\cancel{16}\end{matrix} \Big| \begin{matrix}36\\31\end{matrix} = \begin{matrix}2\\\cancel{80}\\\cancel{1280}\end{matrix} \Big| x,$$

1,280, 80 times. Cancel 16 and 1,280, and write 80 above 1,280. 40 is contained in 80, 2 times. Cancel 40 and 80,

and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 = \$2{,}232$, the same result as was obtained in the last article.

343. Rule.—*Write all the numbers forming the first cause in a vertical column, and draw a vertical line; on the other side of this line write in a vertical column all of the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write, for a fourth column, the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.*

344. EXAMPLE.—If 40 men can dig a ditch 720 feet long, 5 feet wide and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide could 24 men dig in the same time?

SOLUTION.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$\begin{array}{c|c|c|c} 40 & 24 & \begin{matrix}3\\18\\720\end{matrix} & x \\ & = & \begin{matrix}5\\4\end{matrix} & 3 \\ & & & 6 \end{array}$$

whence, $x = 24 \times 5 \times 4 = 480$ feet. Ans.

345. EXAMPLE.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

SOLUTION.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|c|c|c} 10^2 & 16^2 & & 100 & 256 & \\ 20 & 24 & = 1{,}570.8 & x, \text{ or } \begin{matrix}20\\5\end{matrix} & \begin{matrix}24\\6\end{matrix} & = 1{,}570.8 \end{array} \Big| \; x;$$

whence, $x = \dfrac{256 \times 6 \times 1{,}570.8}{5 \times 100} = 4{,}825.4976$ cubic inches. Ans.

346. EXAMPLE.—If a block of granite 8 ft. long, 5 ft. wide and 3 ft. thick weighs 7,200 lb., what will be the weight of a block of granite 12 ft. long, 8 ft. wide and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, we have

$$\begin{array}{c|c|c|c} 8 & \begin{matrix}4\\12\end{matrix} & & \\ 3 & 8 & = 7{,}200 & x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds. Ans.} \\ 3 & 5 & & \end{array}$$

347. EXAMPLE.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines the effects. Hence,

$$\begin{array}{c|c|c|c|c} 3 & 3 & 5 & & 2 \\ 12 & 18 & \begin{matrix}25\\4\end{matrix} & & 64 \\ 30 & x & = 16 & & 12, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days. Ans.} \\ 8 & & & & 4 \\ 10 & 8 & 32 & & 40 \\ & & & & 5 \end{array}$$

348. In examples stated like that in Art. **345**, should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose those numbers which are said to vary inversely from one side of the vertical line to the other side.

EXAMPLE.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect, we have,

$$\begin{array}{c|c|c|c} 15 & 15 & & \\ 20^2 & 24^2 & = 187 & x. \\ 12 & 18 & & \end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c|c|c} 15 & \begin{matrix}15\\2\end{matrix} & & \\ 25 & 36 & = 187 & x, \text{ or } x = \dfrac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds. Ans.} \\ 400 & 576 & & \\ 18 & 12 & & \end{array}$$

EXAMPLES FOR PRACTICE.

349. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each how many rods can 16 men dig in 18 days of 9 hours each?

Ans. 36 rods.

2. If a piece of iron 7 ft. long, 4 in. wide, and 6 in. thick weighs 600 lb., how much will a piece of iron weigh that is 16 ft. long, 8 in. wide and 4 in. thick? Ans. $1,828\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide and 8 feet high, working 8 hours a day? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch? Ans. 204.8 horsepower.

5. Referring to the example in Art. **345,** what will be the volume of a cylinder 20 inches in diameter and 24 inches long?

Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3\times5\times7\times9$ is 945, what is the product of $6\times15\times14\times36$? Ans. 45,360.

NOTICE.

On account of certain changes in the Arithmetic papers, article numbers 340-349 of Arithmetic are repeated in the paper on Mensuration. This will not occasion the student any inconvenience, as there are no references to these article numbers in Mensuration. There is also a slight break in the page numbers, which in no way affects the text matter.

MENSURATION AND USE OF LETTERS IN FORMULAS.

FORMULAS.

340. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note-book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities;" the word quantity is more general in its meaning than the word number.

341. The signs used in formulas are the ordinary signs indicative of operations, and the signs of aggregation. All these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

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The signs indicative of operations are six in number, viz.: $\vdash$, $-$, $\times$, $\div$, $|$, $\surd$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 / 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\surd$) indicates that some root of the quantity to the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\surd 25$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

The signs of aggregation are four in number; viz., ——, ( ), [ ], and { }, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8} \times 7$, $(5+8) \times 7$, $[5+8] \times 7$, $\{5+8\} \times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operations.

While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{(5+8)}$, $\sqrt[3]{[5+8]}$, or $\sqrt[3]{\{5+8\}}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis

is always used. Hence, $(5+8) \times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5) \div 3] \times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5) \div 3] \times 9 - 21\} \div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20-5} \div 3) \times 9 - 21] \div 8\} \times 12$.

As stated in arithmetic, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2+3 \times 4$ equals 14, 3 being multiplied by 4, before adding to 2. Similarly, $10 \div 2 \times 5$ equals 1, since 2×5 equals 10, and $10 \div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5 - 9 \div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be

performed; it is (=), and is called the sign of *equality;* it means that all on one side of it is exactly equal to all on the other side. For example, $2=2$, $5-3=2$, $5\times(14-9)=25$.

342. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam-engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \frac{\text{mean effective pressure (in pounds per square inch)} \times \text{stroke (in feet)} \times \text{area of piston (in square inches)} \times \text{number of strokes (per minute)}}{33{,}000}.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H=\frac{P\times L\times A\times N}{33{,}000},$$

a much simpler and shorter expression. This last expression is called a *formula.*

The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H=\frac{PLAN}{33{,}000}.$$

The sign of multiplication, evidently, can not be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2\cdot3$ means the same as 2×3; $542\cdot749\cdot1{,}006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a\times\sqrt{b+c}$, $3\times(b+c)$, $(b+c)\times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3\times\overline{b+c}$, $\overline{b+c}\times a$, $\sqrt{\overline{b+c}}\times a$, etc., are always written as here printed.

343. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial

letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further, and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned, when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam, at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure, and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure, or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case

primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^a, etc., as they are liable to be mistaken for exponents.

The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body; etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\frac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\frac{1}{4} \times .1138} =$$

$$\frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

344. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\frac{660}{25}}$ sometimes occur. the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\frac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7+\frac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}.$$

Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc., of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the

denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2}=37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1+a^2\,b)^3}$ may also be written $(1+a^2\,b)^{\frac{3}{5}}$, a much simpler expression.

345. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A=\frac{\pi\,r^2\,E}{360}-\frac{c}{2}(r-h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
and h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

SOLUTION.—Applying the formula just given,

$$A=\frac{\pi\,r^2\,E}{360}-\frac{c}{2}(r-h)=\frac{3.1416\times 7.5^2\times 83.46}{360}-\frac{10}{2}(7.5-1.91)=$$

$40.968-27.95=13.018$ square inches, nearly. Ans.

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A=\frac{b}{2}\sqrt{a^2-\left(\frac{a^2+b^2-c^2}{2\,b}\right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then substituting in the formula,

$$A=\frac{b}{2}\sqrt{a^2-\left(\frac{a^2+b^2-c^2}{2b}\right)^2}=\frac{50}{2}\sqrt{21^2-\left(\frac{21^2+50^2-46^2}{2\times 50}\right)^2}$$

$$=\frac{50}{2}\sqrt{441-\left(\frac{441+2{,}500-2{,}116}{100}\right)^2}=25\sqrt{441-\left(\frac{825}{100}\right)^2}$$

$$=25\sqrt{441-8.25^2}=25\sqrt{441-68.0625}=25\sqrt{372.9375}$$

$=25\times 19.312=482.8$ square feet, nearly. Ans.

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P=\frac{SA}{1+q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;
S = ultimate strength (in pounds per square inch) of the material composing the column;
A = area of cross-section of column in square inches;
q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;
l = length of column in inches;
and G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S=150{,}000$, and $q=\frac{1}{25{,}000}$ for flat-ended steel columns; A, the area of the cross-section, $=.7854\,(d_1^2-d_2^2)=.7854\,(14^2-11^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l=20\times 12=240$ inches; and $G^2=\frac{d_1^2+d_2^2}{16}=\frac{14^2+11^2}{16}$. Substituting these values in the formula,

$$P=\frac{SA}{1+q\frac{l^2}{G^2}}=\frac{150{,}000\times .7854\,(14^2-11^2)}{1+\frac{1}{25{,}000}\times\frac{240^2}{\frac{14^2+11^2}{16}}}=$$

$$\frac{150{,}000\times 58.905}{1+.1163}=\frac{8{,}835{,}750}{1.1163}=7{,}915{,}211 \text{ pounds.}\quad \text{Ans.}$$

4. EXAMPLE.—When $A=10$, $B=8$, $C=5$, and $D=4$, what is the value of E in the following:

(*a*) $E=\sqrt[3]{\frac{BCD}{A\left(2+\frac{D^2}{C^2}\right)}}$? (*b*) $E=\frac{A-\frac{3}{4}D+\frac{4B^2}{A+C}}{A-\sqrt{\frac{2B^2}{A+22}}}$?

SOLUTION.—(*a*) Substituting,

$$E=\sqrt[3]{\frac{8\times 5\times 4}{10\left(2+\frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E=\sqrt[3]{\frac{160}{10\left(2+\frac{16}{25}\right)}}=\sqrt[3]{\frac{160}{10\times\frac{66}{25}}}=\sqrt[3]{\frac{\frac{160}{660}}{25}}=\sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E=\sqrt[3]{6.0606}=1.823.\quad \text{Ans.}$$

(*b*) Substituting,

$$E=\frac{10-\frac{3}{4}\times 4+\frac{4\times 8^2}{10+5}}{10-\sqrt{\frac{2\times 8^2}{10+22}}}=\frac{10-3+\frac{4\times 64}{15}}{10-\sqrt{\frac{2\times 64}{32}}}=$$

$$\frac{7+17.066+}{10-\sqrt{4}}=\frac{24.066+}{8}=3.008+.\quad \text{Ans.}$$

345$_1$.—In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example,

let it be required to find how many strokes per minute an engine having a piston area of 78.54 sq. in. must make in order to develop 60 horsepower, if the mean effective pressure is 40 lb. per sq. in., and the length of stroke $1\frac{1}{4}$ ft. By substituting the given values in the formula $H = \frac{PLAN}{33{,}000}$, we have

$$60 = \frac{40 \times 1\frac{1}{4} \times 78.54 \times N}{33{,}000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33{,}000} \times N$, a fraction whose numerator is composed of 3 factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1{,}000} \times N = .119\,N.$$

If 60 equals .119 N, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{m\,v^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4{,}500}{r}.$$

But if 375 equals 4,500 divided by r, then $375 \times r = 4{,}500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4{,}500}{375} = 12$.

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \frac{\frac{2}{3}(A + e)}{c\,e}$. Ans. $x = 1\frac{1}{4}$.

3. $x = \sqrt{\frac{d^2}{2c}} + \sqrt{A\,B^2}$. Ans. $x = 29$.

4. $x = \frac{A\,e}{\sqrt{16\,B\,c}} + \frac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2\,e)\left(\sqrt[3]{B} - \frac{1}{c}\right) + \frac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{13}$.

6. $x = \sqrt{\frac{B\,c\,d}{A\left(2 + \frac{d^3}{e^2}\right)}}$. Ans. $x = .396+$.

MENSURATION.

346. Mensuration treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

347. A straight line is one that does not change its direction throughout its whole length. To distinguish one straight line from another, its two extreme points are designated by letters. The line shown in Fig. 1 would be called the line $A\,B$.

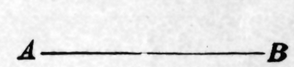

FIG. 1.

348. A **curved line** changes its direction at every point. Curved lines are designated by three or more letters, as the curved line $A\,B\,C$, Fig. 2.

FIG. 2.

349. Parallel lines (Fig. 3) are those which are equally distant from each other at all points.

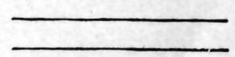
FIG. 3.

350. A line is **perpendicular** to another (see Fig. 4) when it meets that line so as not to incline towards it on either side.

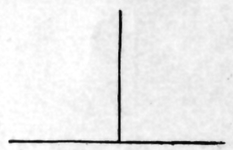
FIG. 4.

351. A **vertical line** is one that points towards the center of the earth, and is also known as a *plumb* line.

352. A **horizontal line** (see Fig. 5) is one that makes a right angle with any vertical line.

FIG. 5.

353. An **angle** is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines $A\,B$ and $C\,B$ is called the angle $A\,B\,C$, or the angle $C\,B\,A$; the letter at the vertex is always placed in the middle. When an angle stands alone so that it can not be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle B.

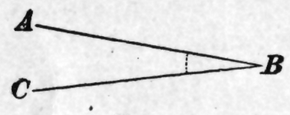

FIG. 6.

354. If one straight line meets another straight line at a point between its ends, as in Fig. 7, two angles, $A\,B\,C$ and $A\,B\,D$, are formed, which are called **adjacent angles.**

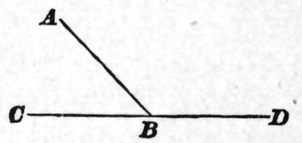

FIG. 7.

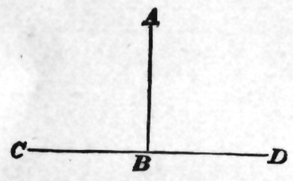

FIG. 8.

355. When these adjacent angles, $A\,B\,C$ and $A\,B\,D$, are equal, as in Fig. 8, they are called **right angles.**

356. An **acute angle** is less than a right angle. $A\,B\,C$, Fig. 9, is an acute angle.

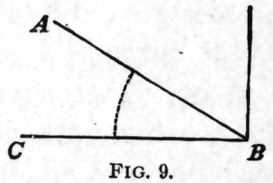

FIG. 9.

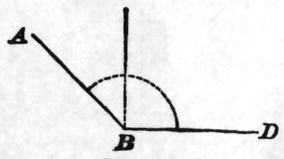

FIG. 10.

357. An **obtuse angle** is greater than a right angle. $A\,B\,D$ (Fig. 10) is an obtuse angle.

358. A **circle** (see Fig. 11) is a figure bounded by a curved line, called the **circumference**, every point of which is equally distant from a point within, called the **center.**

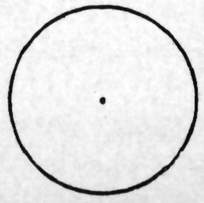
FIG. 11.

359. An **arc** of a circle is any part of its circumference; thus $a\,e\,b$, Fig. 12, is an arc of the circle.

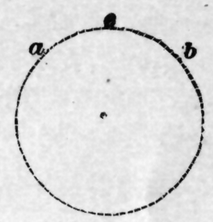

FIG. 12.

The circumference of every circle is considered to be divided into 360 equal parts, or arcs, called **degrees;** every degree is subdivided into 60 equal parts, called **minutes,** and every minute is again divided into 60 equal parts, called **seconds.**

Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of a degree will be different in circles of different sizes, but the proportion of the length of an arc of one degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Degrees, minutes, and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″ is read 37 degrees, 14 minutes, and 44 seconds.

360. The arcs of circles are used to measure angles. An angle having its vertex at the center of a circle is measured by the arc included between its sides; thus, in Fig. 13, the arc $F\,B$ measures the angle $F\,O\,B$. If the arc $F\,B$ contains 20°, or $\frac{20}{360}$ of the circumference, the angle $F\,O\,B$ would be an angle of 20°; if it contained 20° 14′ 18″, it would be an angle of 20° 14′ 18″, etc.

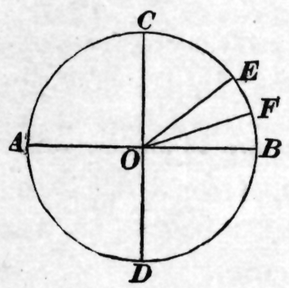

FIG. 13.

In the figure, if the line $C\,D$ be drawn perpendicular to $A\,B$, the adjacent angles will be equal, and the circle will be divided into four equal angles, each of which will be a right angle. A right angle, therefore, is an angle of $\frac{360°}{4}$, or 90°; two right angles are measured by 180°, or half the circumference, and four right angles by the whole circumference, or 360°. One-half of a right angle, as $E\,O\,B$, is an

angle of 45°. An *acute* angle may now be defined as an angle of *less* than 90°, and an *obtuse* angle as one of *more* than 90°. These values are important, and should be remembered.

361. From the foregoing it will be evident that if a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles, or 180°. Thus, in Fig. 14, angles $COB + DOC + EOD + FOE + AOF$ = 2 right angles, or 180°.

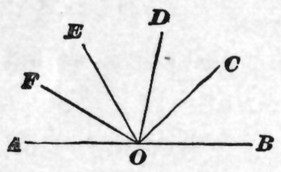

FIG. 14.

362. Also, if through a given point any number of straight lines be drawn, the sum of all the angles formed about the points of intersection equals four right angles, or 360°. Thus, in Fig. 15, angles $HOF + FOC + COA + AOG + GOE + EOD + DOB + BOH$ = four right angles, or 360°.

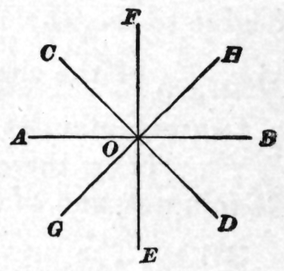

FIG. 15.

EXAMPLE.—In a fly-wheel, with 12 arms, how many degrees are there in the angle included between the center lines of any two arms, the arms being spaced equally?

SOLUTION.—Since there are 12 arms, there are 12 angles, which together equal 360°. Hence, one angle equals $\frac{1}{12}$ of 360°, or $\frac{360°}{12} = 30°$. Ans.

EXAMPLES FOR PRACTICE.

1. How many seconds in 32° 14′ 6″? Ans. 116,046 sec.
2. How many degrees, minutes, and seconds do 38,582 seconds amount to? Ans. 10° 43′ 2″.
3. How many right angles are there in an angle of 170°? Ans. $1\frac{8}{9}$ right angles.
4. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{4}{5}$ of a right angle.
5. If one straight line meets another so as to form an angle of 20° 10′, what does its adjacent angle equal? Ans. 159° 50′.
6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°.

QUADRILATERALS.

363. A **plane figure** is any part of a plane or flat surface bounded by straight or curved lines.

364. A **quadrilateral** is a plane figure bounded by four straight lines.

365. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

366. A **rectangle** (Fig. 16) is a parallelogram whose angles are all right angles.

FIG. 16.

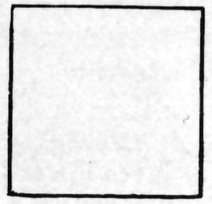
FIG. 17.

367. A **square** (Fig. 17) is a rectangle whose sides are all of the same length.

368. A **rhomboid** (Fig. 18) is a parallelogram whose opposite sides are equal and parallel, and whose angles are not right angles.

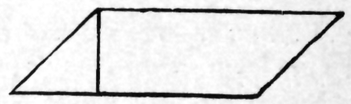
FIG. 18.

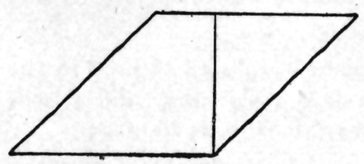
FIG. 19.

369. A **rhombus** (Fig. 19) is a parallelogram having equal sides, and whose angles are not right angles.

370. A **trapezoid** (Fig. 20) is a quadrilateral which has only two of its sides parallel.

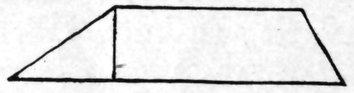
FIG. 20.

371. The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between the parallel lines, as shown by the dotted lines in Figs. 18, 19, and 20.

372. The **base** of *any* plane figure is the side on which it is supposed to stand.

373. The **area** of a surface is expressed by the number of unit squares it will contain.

374. A **unit square** is the square having a unit for its side. For example, if the unit is 1 inch, the unit square is the square each of whose sides measures 1 inch in length, and the area of a surface would be expressed by the number of square inches it would contain. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area of the given surface would be the number of square feet it would contain, etc.

The square that measures one inch on a side is called a **square inch,** and the one that measures one foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft.

375. To find the area of any parallelogram:

Rule 44.—*Multiply the base by the altitude.*

NOTE.—Before multiplying, the base and altitude must be reduced to the same kind of units; that is, if the base should be given in feet and the altitude in inches, they could not be multiplied together until either the altitude had been reduced to feet, or the base to inches. This principle holds throughout the subject of mensuration.

EXAMPLE.—The sides of a square piece of sheet iron are each $10\frac{1}{4}$ inches long. How many square inches does it contain?

SOLUTION.—$10\frac{1}{4}$ inches = 10.25 inches when reduced to a decimal. The base and altitude are each 10.25 inches. Multiplying them together, $10.25 \times 10.25 = 105.06+$ sq. in. Ans.

EXAMPLE.—What is the area in square rods of a piece of land in the shape of a rhomboid, one side of which is 8 rods long, and whose length, measured on a line perpendicular to this side, is 200 feet?

SOLUTION.—The base is 8 rods and the altitude 200 feet. As the answer is to be in rods, the 200 feet should be reduced to rods. Reducing $200 \div 16\frac{1}{2} = 200 \div \frac{33}{2} = 12.12$ rods. Hence, area $= 8 \times 12.12 = 96.96$ sq. rd. Ans.

376. To find the area of a trapezoid:

Rule 45.—*Multiply one-half the sum of the parallel sides by the altitude.*

EXAMPLE.—A board 14 feet long is 20 inches wide at one end and 16 inches wide at the other. If the ends are parallel, how many square feet does the board contain?

SOLUTION.—One-half the sum of the parallel sides $= \frac{20+16}{2} = 18$ inches $= 1\frac{1}{2}$ feet. The length of the board corresponds to the altitude of a trapezoid. Hence, $14 \times 1\frac{1}{2} = 21$ sq. ft. Ans.

377. Having given the area of a parallelogram and one dimension, to find the other dimension:

Rule 46.—*Divide the area by the given dimension.*

EXAMPLE.—What is the width of a parallelogram whose area is 212 square feet and whose length is $26\frac{1}{2}$ feet?

SOLUTION.—$212 \div 26\frac{1}{2} = 212 \div \frac{53}{2} = 8$ feet. Ans.

The following examples illustrate a few special cases:

EXAMPLE.—An engine room is 22 feet by 32 feet. The engine-bed occupies a space of 3 feet by 12 feet; the fly-wheel pit, a space of 2 feet by 6 feet, and the outer bearing, a space of 2 feet by 4 feet. How many square feet of flooring will be required for the room?

SOLUTION.—Area of engine-bed $= 3 \times 12 = 36$ sq. ft.
Area of fly-wheel pit $= 2 \times 6 = 12$ sq. ft.
Area of outer bearing $= 2 \times 4 = 8$ sq. ft.
Total, 56 sq. ft.
Area of engine room $= 22 \times 32 = 704$ sq. ft.
$704 - 56 = 648$ square feet of flooring required. Ans.

EXAMPLE.—How many square yards of plaster will it take to cover the sides and ceiling of a room 16×20 feet and 11 feet high, having four windows, each 7×4 feet, and three doors, each 9×4 feet over all, the baseboard coming 6 inches above the floor?

SOLUTION.—
Area of ceiling $= 16 \times 20 =$ 320 sq. ft.
Area of end walls $= 2(16 \times 11) =$ 352 sq. ft.
Area of side walls $= 2(20 \times 11) =$ 440 sq. ft.
Total area = 1,112 sq. ft.

From the above must be deducted:
Windows $= 4(7 \times 4) = 112$ sq. ft.
Doors $= 3(9 \times 4) = 108$ sq. ft.

Baseboard less the width of three doors $= (72 - 12) \times \frac{6}{12} = 30$ sq. ft.

Total number of feet to be deducted $= 112 + 108 + 30 = 250$ sq. ft.

Hence, number of square feet to be plastered $= 1{,}112 - 250 = 862$ sq. ft., or $95\frac{7}{9}$ sq. yd. Ans.

EXAMPLE.—How many acres are contained in a rectangular tract of land 800 rods long and 520 rods wide?

SOLUTION.—$800 \times 520 = 416{,}000$ sq. rd. Since there are 160 square rods in one acre, the number of acres $= 416{,}000 \div 160 = 2{,}600$ acres. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches, and whose altitude is 3 feet? Ans. 21 sq. ft.

2. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roof? Ans. 1,840 lb.

3. One side of a room measures 16 ft. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. How many square feet in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

5. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case.

6. How many square yards of plastering will be required for the ceiling and walls of a room 10×15 feet and 9 feet high; the room contains one door $3\frac{1}{2} \times 7$ feet, three windows $3\frac{1}{2} \times 6$ feet, and a baseboard 8 inches high? Ans. 53.5 sq. yd.

THE TRIANGLE.

378. A **triangle** is a plane figure having three sides.

FIG. 21.

379. An **isosceles** triangle is one having two of its sides equal; see Fig. 21.

380. An **equilateral** triangle (Fig. 22) is one having all of its sides of the same length.

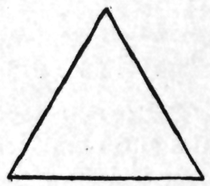

FIG. 22.

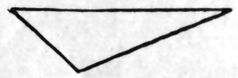

FIG. 23.

381. A **scalene** triangle (Fig. 23) is one having no two of its sides equal.

382. A **right-angled** triangle (Fig. 24) is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.**

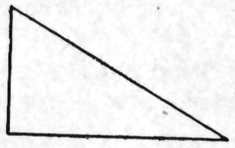

FIG. 24.

In any triangle the sum of the three angles equals two right angles, or 180°. Thus, in Fig. 25, the sum of the angles A, B, and C equals two right angles, or 180°. Hence, if any two angles of a triangle are given and it is required to find the third angle:

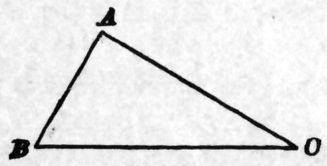

FIG. 25.

Rule 47.—*Add together the two given angles, and subtract their sum from 180°; the result will be the third angle.*

EXAMPLE.—If two angles of a triangle = 48° 16′ and 47° 50′ respectively, what does the third angle equal?

SOLUTION.—First reduce 48° 16′ and 47° 50′ to minutes, for convenience in adding and subtracting the angles. $48° = 48 \times 60 = 2{,}880'$; $2{,}880' + 16' = 2{,}896'$; hence, $48° 16' = 2{,}896'$. In like manner, $47° 50' = 47 \times 60' = 2{,}820' + 50' = 2{,}870'$. Adding the two angles together, and subtracting from 180° reduced to minutes, $2{,}896' + 2{,}870' = 5{,}766'$; $180° = 180 \times 60 = 10{,}800'$; $10{,}800 - 5{,}766 = 5{,}034'$. Reducing this last number to degrees and minutes, $\frac{5{,}034}{60} = 83\frac{54}{60}° = 83° 54'$. Hence, the third angle in the triangle = 83° 54′. Ans.

383. In any right-angled triangle there can be but one right angle, and since the sum of all the angles is two right angles, it is evident that the sum of the two acute angles must equal one right angle, or 90°. Therefore, if in any right-angled triangle one acute angle is known, to find the other acute angle:

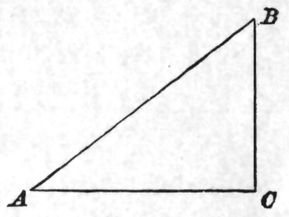

FIG. 26.

Rule 48.—*Subtract the known acute angle from 90°; the result will be the other acute angle.*

EXAMPLE.—If one acute angle, as A, of the right-angled triangle $A\,B\,C$, Fig. 26, equals 30°, what does the angle B equal?

SOLUTION.—$90° - 30° = 60°$. Ans.

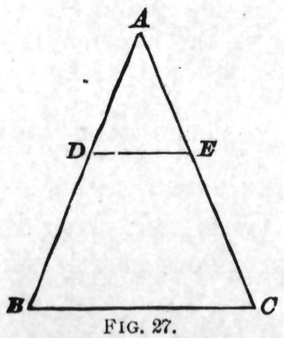

FIG. 27.

384. If a straight line be drawn through two sides of a triangle, parallel to the third side, a second triangle will be formed whose sides will be proportional to the corresponding sides of the first triangle. Thus, in the triangle $A\,B\,C$, Fig. 27, if the line $D\,E$ be drawn parallel to the side $B\,C$, the triangle $A\,D\,E$ will be formed and we shall have

(1) Side $A\,D$: side $D\,E$ = side $A\,B$: side $B\,C$; and,

(2) Side $A\,E$: side $D\,E$ = side $A\,C$: side $B\,C$; also,

(3) Side $A\,D$: side $A\,E$ = side $A\,B$: side $A\,C$.

EXAMPLE.—In Fig. 27, if $A\,B = 24$, $B\,C = 18$, and $D\,E = 8$, what does $A\,D$ equal?

SOLUTION.—Writing these values for the sides in (1),

$A\,D : 8 = 24 : 18$; whence $A\,D = \frac{24 \times 8}{18} = 10\frac{2}{3}$. Ans.

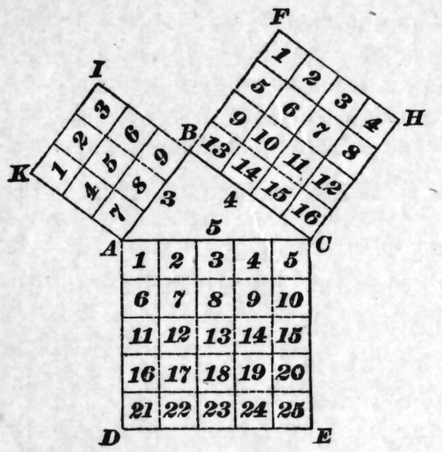

FIG. 28.

385. In any right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If $A\,B\,C$, Fig. 28, is a right-angled triangle, right-angled at B, then the square described upon the hypotenuse $A\,C$ is equal to the sum of the squares described upon the sides $A\,B$ and $B\,C$. Hence, having given the two sides forming the right angle in a right-angled triangle, to find the hypotenuse:

Rule 49.—*Square each of the sides forming the right angle; add the squares together, and take the square root of the sum.*

EXAMPLE.—If $A\,B = 3$ inches and $B\,C = 4$ inches, what is the length of the hypotenuse $A\,C$?

SOLUTION.—Squaring each of the given sides, $3^2 = 9$ and $4^2 = 16$. Taking the square root of the sum of 9 and 16, the hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$ inches. Ans.

If the hypotenuse and one side are given, the other side can be found as follows:

Rule 50.—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

EXAMPLE.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

SOLUTION.—$3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and the $\sqrt{16} = 4$ inches. Ans.

EXAMPLE.—If from a church steeple which is 150 feet high, a rope is to be attached to the top, and to a stake in the ground, which is 85 feet from the center of the base (the ground being supposed to be level), what must be the length of the rope?

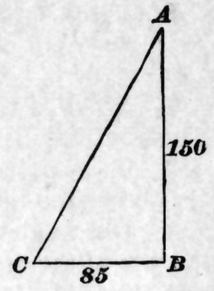

FIG. 29.

SOLUTION.—In Fig. 29, $A\,B$ represents the steeple, 150 feet high; C a stake 85 feet from the foot of the steeple, and $A\,C$ the rope. Here we have a right-angled triangle, right-angled at B, and $A\,C$ is the hypotenuse. The square of $A\,B = 150^2 = 22{,}500$; of $C\,B$, $85^2 = 7{,}225$. $22{,}500 + 7{,}225 = 29{,}725$; $\sqrt{29{,}725} = 172.4$ feet, nearly. Ans.

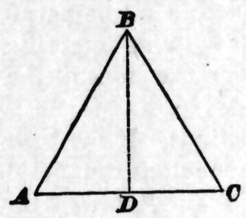

FIG. 30.

386. The **altitude** of any triangle is a line, as $B\,D$, drawn from the vertex B of the angle opposite the base $A\,C$, perpendicular to the base, as in Fig. 30, or to the base extended, as in Fig. 31.

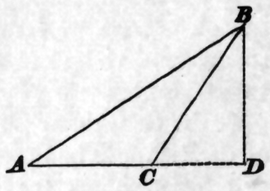

FIG. 31.

If in any parallelogram a straight line, called the **diagonal,** be drawn, connecting two opposite corners it will divide the parallelogram into two equal triangles, as $A\,D\,B$ and $D\,B\,C$ in Fig. 32. The area of each triangle will equal one-half the area of the parallelogram, or one-half the product of the base and the altitude. Hence, to find the area of any triangle:

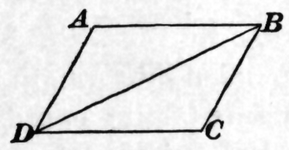

FIG. 32.

Rule 51.—*Multiply the base by the altitude, and divide the product by 2.*

EXAMPLE.—What is the area in square feet of a triangle whose base is 18 feet, and whose altitude is 7 feet 9 inches?

SOLUTION.—7 feet 9 inches $= 7\frac{3}{4}$ feet $= \frac{31}{4}$ feet. $18 \times \frac{31}{4} = 139\frac{1}{2}$, and one-half of $139\frac{1}{2} = 69\frac{3}{4}$ square feet. Ans.

To find the altitude or base of a triangle, having given the area and the base or altitude:

Rule 52.—*Multiply the area by 2, and divide by the given dimension.*

EXAMPLE.—What must be the height of a triangular piece of sheet metal to contain 100 square inches, if the base is 10 inches long?

SOLUTION.—$100 \times 2 = 200$; $200 \div 10 = 20$ inches. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose base is 18 feet long, and whose altitude is 10 feet 6 inches? Ans. 94.5 sq. ft.

2. Two angles of a scalene triangle together equal 100° 4′. What is the size of the third angle? Ans. 79° 56′.

3. One angle of a right-angled triangle equals 20° 10′ 5″. What is the size of the other acute angle? Ans. 69° 49′ 55″.

4. A ladder 65 feet long reaches to the top of a wall when its foot is 25 feet from the wall. How high is the wall? Ans. 60 ft.

5. Draw a triangle, and through two of its sides draw a line parallel to the base. Letter the different lines, and then, without referring to the text, write out the proportions existing between the sides of the two triangles.

6. A triangular piece of sheet metal weighs 24 pounds. If the base of the triangle is 4 feet and its height 6 feet, how much does the metal weigh per square foot? Ans. 2 lb.

7. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

8. Two sides of a right-angled triangle are 92 feet and 69 feet long. How long is the hypotenuse? Ans. 115 ft.

POLYGONS.

387. A **polygon** is a plane figure bounded by straight lines. The term is usually applied to a figure having more than four sides. The bounding lines are called the **sides,** and the sum of the lengths of all the sides is called the **perimeter** of the polygon.

388. A **regular polygon** is one in which all of the sides and all of the angles are equal.

389. A polygon of five sides is called a **pentagon;** one with six sides a **hexagon;** one of seven sides a **heptagon,** etc. Regular polygons having from five to twelve sides are shown in Fig. 33. In any polygon, the sum of all

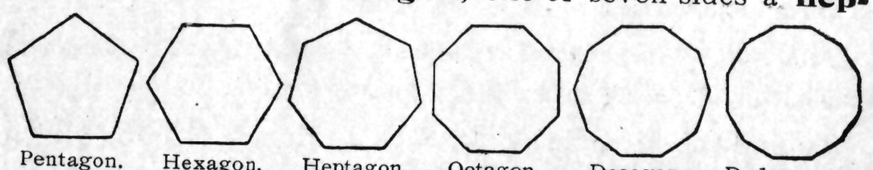

FIG. 33.

the interior angles, as $A + B + C + D + E$, Fig. 34, equals 180° multiplied by a number which is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a *regular* polygon:

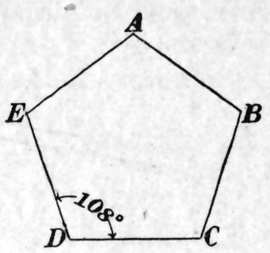

FIG. 34.

Rule 53.—*Multiply 180° by the number of sides less two, and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle.*

EXAMPLE.—If Fig. 34 is a regular pentagon, how many degrees are there in each interior angle?

SOLUTION.—In a pentagon there are five sides; hence, $5 - 2 = 3$ and $180 \times 3 = 540$; $540 \div 5 = 108°$ in each angle. Ans.

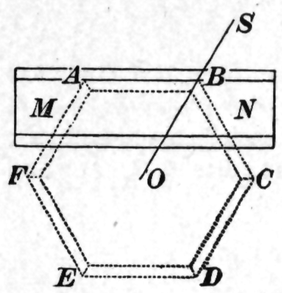

FIG. 35.

EXAMPLE.—It is desired to make a miter-box in which to cut a strip of molding to fit around a column having the shape of a regular hexagon. At what angle should the saw run across the miter-box?

SOLUTION.—In Fig. 35, let AB, BC, CD, etc., represent the pieces of molding as they will fit around the column. First find the size of one of the equal angles of the polygon by the above rule. Number of sides $= 6$; $6 - 2 = 4$; hence, $180 \times 4 = 720$, and $720 \div 6 = 120°$ in each angle. Now, let MN represent the miter-box, and OS the direction in which the saw should run; then, ABO is the angle made by the saw with the side of the miter-box; but, as the polygon is a regular one, this angle is one-half the interior angle ABC, which we have found to be 120°. Hence, the saw should run at an angle of $\frac{120}{2} = 60°$ with the side of the miter-box.

390. The area of any regular polygon may be found by drawing lines from the center to each angle, and computing the area of each triangle thus formed. Hence, to find the area of any regular polygon:

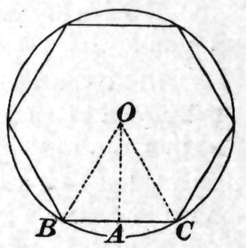

FIG. 36.

Rule 54.—*Multiply the length of a side by half the distance from the side to the center, and that product by the number of sides. The last product will be the area of the figure.*

EXAMPLE.—In Fig. 36 the side BC of the regular hexagon is 12 inches and the distance AO is 10.4 inches; required, the area of the polygon.

SOLUTION.—$10.4 \div 2 = 5.2$; $12 \times 5.2 \times 6 = 374.4$ sq. in. Ans.

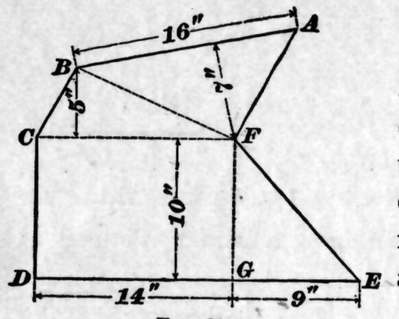

FIG. 37.

391. To obtain the area of any irregular polygon, draw diagonals dividing the polygon into triangles and quadrilaterals, and compute the areas of these separately; their sum will be the area of the figure.

EXAMPLE.—It is required to find the area of the polygon $ABCDEF$, Fig. 37.

SOLUTION.—Draw the diagonals BF and CF and the line FG perpendicular to DE, dividing the figure into the triangles ABF, BCF, and FGE, and the rectangle $FCDG$. Let it be supposed that the altitudes of the figures and the lengths of the sides AB, DG, and GE are as indicated in the polygon above. Then,

$$\text{Area } ABF = \frac{16 \times 7}{2} = 56 \text{ sq. in.}$$

$$\text{Area } BCF = \frac{14 \times 5}{2} = 35 \text{ sq. in.}$$

$$\text{Area } FCDG = 14 \times 10 = 140 \text{ sq. in.}$$

$$\text{Area } FGE = \frac{9 \times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56 + 35 + 140 + 45 = 276$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. How many degrees are there in one of the angles of a regular octagon? Ans. 135°.

2. Find the area of the polygon $ABCDEF$ (see Fig. 37), supposing each of the given dimensions to be increased to $1\frac{1}{2}$ times the length given in the figure. Ans. 621 sq. in.

3. What is the area of a regular heptagon, whose sides are 4 inches long, the distance from one side to the center being 4.15 inches? Ans. 58.1 sq. in.

4. At what angle should the saw run in a miter-box to cut strips to fit around the edge of a table top, made in the shape of a regular pentagon? Ans. 54°.

THE CIRCLE.

FIG. 38.

392. A **circle** (Fig. 38) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

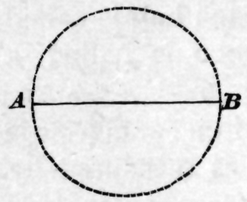

FIG. 39.

393. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference; thus, AB (Fig. 39) is a diameter of the circle.

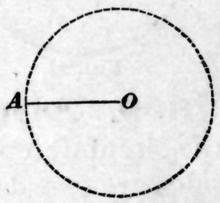

FIG. 40.

394. The **radius** of a circle, AO (Fig. 40), is a straight line drawn from the center O to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and all radii of a circle are equal.

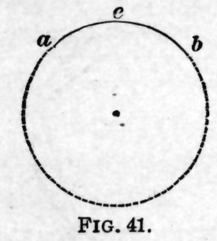

FIG. 41.

395. An **arc** of a circle (see $a\ e\ b$, Fig. 41) is any part of its circumference.

396. A **chord** is a straight line joining any two points in a circumference; or it is a straight line joining the extremities of an arc; thus, the straight line $a\ b$, Fig. 42, is a chord of the circle whose corresponding arc is $a\ e\ b$.

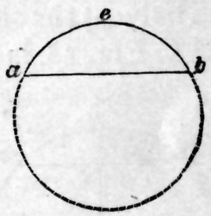

FIG. 42.

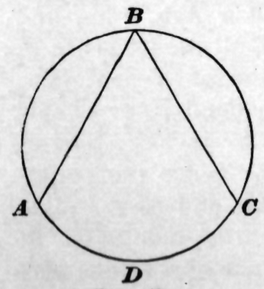

FIG. 43.

397. An **inscribed angle** is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 43, ABC is an inscribed angle, and it is measured by one-half the arc ADC.

EXAMPLE.—If in Fig. 43, the arc $A\,D\,C = \frac{2}{5}$ of the circumference, what is the measurement of the inscribed angle $A\,B\,C$?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference $= 360°$; hence, $360° \times \frac{1}{5} = 72°$; therefore, angle $A\,B\,C$ is an angle of $72°$.

398. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.**

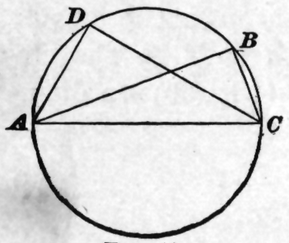

FIG. 44.

Any angle inscribed in a semicircle is a right angle, since it is measured by one-half a semi-circumference, or $180° \div 2 = 90°$. Thus, the angles $A\,D\,C$ and $A\,B\,C$, Fig. 44, are right angles, since they are inscribed in a semicircle.

399. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle, and whose sides are chords, as $A\,B\,C\,D\,E$, Fig. 45.

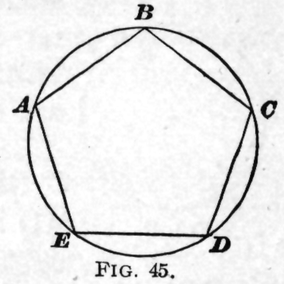

FIG. 45.

The sides of an inscribed regular hexagon have the same length as the radius of the circle.

If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\,C$, Fig. 46, is perpendicular to the chord $A\,B$, $A\,D = D\,B$.

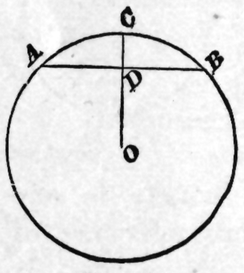

FIG. 46.

EXAMPLE.—If a regular pentagon be inscribed in a circle, and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

SOLUTION.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths; the perimeter of a pentagon, which equals the distance around it, or equals the sum of all the sides, is 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, which equals the length of each of the parts of the side, cut by a radius perpendicular to it.

400. If, from any point on the circumference of a circle, a perpendicular be let fall upon a given diameter, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other. That is, the perpendicular will be a *mean proportional* between the two parts.

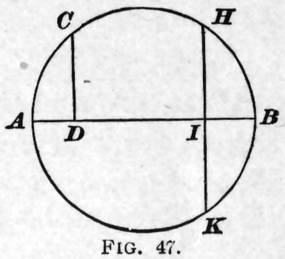

FIG. 47.

If $A\,B$, Fig. 47, is the given diameter, and C any point on the circumference, then $A\,D : C\,D :: C\,D : D\,B$, $C\,D$ being a mean proportional between $A\,D$ and $D\,B$.

EXAMPLE.—If $H\,K = 30$ feet, and $I\,B = 8$ feet, what is the diameter of the circle, $H\,K$ being perpendicular to $A\,B$?

SOLUTION.—30 feet $\div$ 2 feet $= 15$ feet $= I\,H$. And $B\,I : I\,H :: I\,H : I\,A$, or $8 : 15 :: 15 : I\,A$.

Therefore, $I\,A = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet, and $I\,A + I\,B = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= A\,B$ the diameter of the circle. Ans.

401. When the diameter of a circle and the lengths of the two parts into which it is divided are given, the length of the perpendicular may be found by multiplying the lengths of the two parts together and extracting the square root of the product.

EXAMPLE.—In Fig. 47, the diameter of the circle $A\,B$ is $36\frac{1}{8}$ feet, and the distance $B\,I$ is 8 feet; what is the length of the line $H\,K$?

SOLUTION.—As the diameter of the circle is $36\frac{1}{8}$ feet, and as $B\,I$ is 8 feet, $I\,A$ is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. The two parts, therefore, are 8 and $28\frac{1}{8}$ feet, and their product $= 8 \times 28\frac{1}{8} = 8 \times \frac{225}{8} = 225$; the square root of their product $= \sqrt{225} = 15$ feet, and as $H\,K = I\,H + I\,K$, or $2\,I\,H$, $H\,K = 15 \times 2 = 30$ feet. Ans.

402. To find the circumference of a circle, the diameter being given:

Rule 55.—*Multiply the diameter by 3.1416.*

EXAMPLE.—What is the circumference of a circle whose diameter is 15 inches?

SOLUTION.—$15 \times 3.1416 = 47.124$ inches. Ans.

403. To find the diameter of a circle, the circumference being given:

Rule 56.—*Divide the circumference by 3.1416.*

EXAMPLE.—What is the diameter of a circle whose circumference is 65.973 inches?

SOLUTION.—$65.973 \div 3.1416 = 21$ inches. Ans.

404. To find the length of an arc of a circle:

Rule 57.—*Multiply the length of the circumference of the circle of which the arc is a part by the number of degrees in the arc, and divide by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the arc being 18 in.?

SOLUTION.—$18 \times 2 = 36$ in. = the diameter of the circle. $36 \times 3.1416 = 113.1$ in., the circumference of the circle of which the arc is a part.

$113.1 \times \frac{24}{360} = 7.54$ in., or the length of the arc. Ans.

405. To find the area of a circle:

Rule 58.—*Square the diameter, and multiply by .7854.*

EXAMPLE.—What is the area of a circle whose diameter is 15 inches?

SOLUTION.—$15^2 = 225$; and $225 \times .7854 = 176.72$ sq. in. Ans.

406. Given the area of a circle, to find its diameter:

Rule 59.—*Divide the area by .7854, and extract the square root of the quotient.*

EXAMPLE.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

SOLUTION.—$\sqrt{\frac{17,671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, or the diameter. Ans.

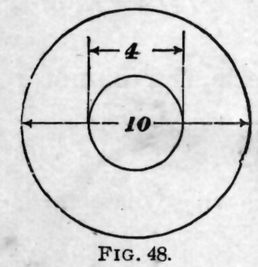

FIG. 48.

EXAMPLE.—What is the area of a flat circular ring, Fig. 48, whose outside diameter is 10 inches, and whose inside diameter is 4 inches?

SOLUTION.—The area of the large circle = $10^2 \times .7854 = 78.54$ square inches; the area of the small circle = $4^2 \times .7854 = 12.57$ square inches. The area of the ring is the difference between these areas, or $78.54 - 12.57 = 65.97$ square inches. Ans.

407. To find the area of a sector (a **sector** of a circle is the area included between two radii and the circumference, as, for example, the area *C O E*, Fig. 13):

Rule 60.—*Divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

EXAMPLE.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.—$\frac{75}{360} = \frac{5}{24}$; and $12^2 \times .7854 = 113.1$ square inches.

$113.1 \times \frac{5}{24} = 23.56$ square inches, the area. Ans.

408. To find the area of a segment of a circle (a **segment** of a circle is the area included between a chord and its arc; for example, the area *A B C*, Fig. 49):

Rule 61.—*Divide the diameter by the height of the segment; subtract .608 from the quotient, and extract the square root of the remainder. This result multiplied by 4 times the square of the height of the segment, and then divided by 3, will give the area, very nearly.*

The rule, expressed as a formula, is as follows, where D = the diameter of the circle, and h = the height of the segment, see Fig. 49:

$$\text{Area of } A\,B\,C\,A = \frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}.$$

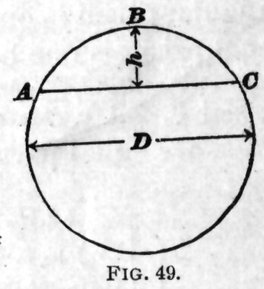

FIG. 49.

EXAMPLE.—What is the area of the segment of a circle whose diameter is 54 inches, the height of the segment being 20 inches?

SOLUTION.—Substituting in the formula,

$$\text{Area} = \frac{4 \times 20^2}{3}\sqrt{\frac{54}{20} - .608}.$$

$\frac{4 \times 20^2}{3} = \frac{4 \times 400}{3} = \frac{1,600}{3}$; $\sqrt{\frac{54}{20} - .608} = \sqrt{2.092} = 1.447$; $\frac{1,600}{3} \times 1.447 = 771.7$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. An angle inscribed in a circle intercepts one-third of the circumference. How many degrees are there in the angle? Ans. 60°

2. Suppose that in Fig. 47, the diameter $A\,B = 15$ feet, and the distance $B\,I = 3$ feet. What is the length of the line $H\,K$?

Ans. 12 ft.

3. The diameter of a fly-wheel is 18 feet. What is the distance around it to the nearest 16th of an inch? Ans. 56 ft. $6\frac{9}{16}$ in.

4. A carriage wheel was observed to make $71\frac{2}{3}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

5. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

6. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

7. What must be the diameter of a circle to contain 100 square inches? Ans. 11.28 in.

8. Compute the area of a segment, whose height is 11 inches, and the radius of whose arc is 21 inches. Ans. 289.04 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

THE PRISM AND CYLINDER.

409. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their intersections are called **edges.**

410. A **prism** is a solid whose ends are equal and parallel polygons, and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base; a hexagonal prism is one having a hexagon for its base, etc.

411. A **cylinder** is a body of uniform diameter whose ends are equal parallel circles.

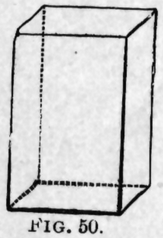

FIG. 50.

412. A **parallelopipedon** (Fig. 50) is a prism whose bases (ends) are parallelograms.

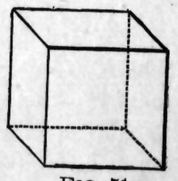

FIG. 51.

413. A **cube** (Fig. 51) is a prism whose faces and ends are squares. All the faces of a cube are equal.

In the case of plane figures, we have had to do with perimeters and areas. In the case of solids, we have to do with the areas of their outside surfaces, and with their contents or volumes.

414. The **entire surface** of any solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

415. A **unit of volume** is a cube each of whose edges is equal in length to the unit. The **volume** is expressed by the number of times it will contain a *unit of volume.*

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges each measure 1 inch, this cube being 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

Instead of the word *volume*, the expression **cubical contents** is sometimes used.

416. To find the area of the convex surface of a prism or cylinder:

Rule 62.—*Multiply the perimeter of the base by the altitude.*

EXAMPLE.—A block of marble is 24 inches long, and its ends are 9 inches square. What is the area of its convex surface?

SOLUTION.—$9 \times 4 = 36$ = the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1{,}026$ sq. in. Ans.

EXAMPLE.—How many square feet of sheet iron will be required for a pipe $1\frac{1}{2}$ feet in diameter and 10 feet long, neglecting the amount necessary for lapping?

SOLUTION.—The problem is to find the convex surface of a cylinder $1\frac{1}{2}$ feet in diameter and 10 feet long. The perimeter, or circumference, of the base $= 1\frac{1}{2} \times 3.1416 = 1.5 \times 3.1416 = 4.712$ feet. The convex surface $= 4.712 \times 10 = 47.12$ sq. ft. of metal. Ans.

417. To find the volume of a prism or a cylinder:

Rule 63.—*Multiply the area of the base by the altitude.*

EXAMPLE.—What is the weight of a length of wrought-iron shafting 16 feet long and 2 inches in diameter? Wrought iron weighs .28 pound per cubic inch.

SOLUTION.—The shaft is a cylinder 16 feet long. The area of one end, or the base, $= 2^2 \times .7854 = 3.1416$ sq. in. Since the weight of the iron is given per cubic inch, the contents of the shaft must be found in cubic inches. The length, 16 feet, reduced to inches $= 16 \times 12 =$ 192 in.; $3.1416 \times 192 = 603.19$ cu. in. = the volume. The weight $=$ $603.19 \times .28 = 168.89$ lb. Ans.

EXAMPLE.—Find the cubical contents of a hexagonal prism, Fig. 52, 12 inches long, each edge of the base being one inch long.

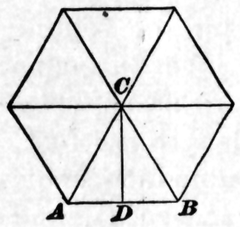

FIG. 52.

SOLUTION.—In order to obtain the area of one end, the distance $C\,D$ from the center C to one side must be found.

In the right-angled triangle $C\,D\,A$, side $A\,D = \frac{1}{2}\,A\,B$, or one-half inch, and since the polygon is a hexagon, side $C\,A$ = distance $A\,B$, or one inch. Hence, $C\,A$ being the hypotenuse, the length of side $C\,D = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{1^2 - .5^2} = \sqrt{.75}$, or .866 inch. Area of triangle $A\,C\,B = \frac{1 \times .866}{2} = .433$ sq. in.; area of the whole polygon $= .433 \times 6 = 2.598$ sq. in. Hence, the contents of the prism $= 2.598 \times 12 = 31.176$ cu. in. Ans.

EXAMPLE.—It is required to find the number of cubic feet of steam space in the boiler shown in Fig. 53. The boiler is 16 feet long between

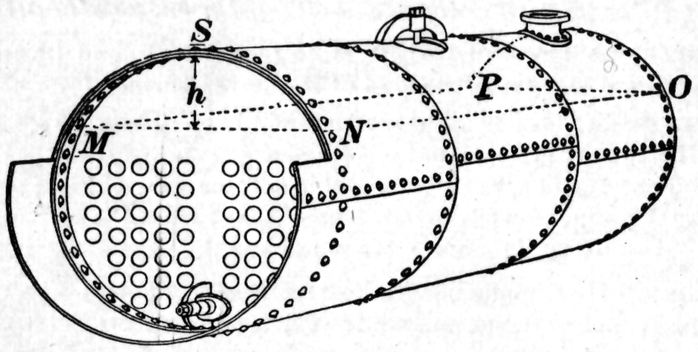

FIG. 53.

heads, 54 inches in diameter, and the mean water line $M\,N$ is at a distance of 16 inches from the top of the boiler. The volume of the steam outlet casting may be neglected.

SOLUTION.—The volume of the steam space, which is that space within the boiler above the surface $M\,N\,O\,P$ of the water, is found by the rule for finding the volume of a prism or cylinder, the area $M\,N\,S$ being the base, and the length $N\,O$ the altitude. First obtain the area of the segment $M\,N\,S$, whose height h is 16 inches, in square feet; then multiply the result by 16, the length of the boiler.

By the formula previously given, the area of the segment $=$

$$\frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608} = \frac{4 \times 16^2}{3}\sqrt{\frac{54}{16} - .608};$$

$$\frac{4 \times 16^2}{3} = 341.33;\ \sqrt{\frac{54}{16} - .608} = \sqrt{2.767} = 1.663.$$

Hence, the area $= 341.33 \times 1.663 = 567.63$ sq. in. This, reduced to square feet, $= 567.63 \div 144 = 3.942$ sq. ft., and the volume, therefore, $=$ $3.942 \times 16 = 63.07$ cu. ft. Ans.

In the above solution, the space occupied by the stays is not considered, for sake of simplicity. They are not shown in the figure.

EXAMPLE.—In the above boiler there are 60 tubes, $3\frac{1}{4}$ inches outside diameter. How many gallons of water will it take to fill the boiler up to the mean water level, there being 231 cubic inches in a gallon?

SOLUTION.—Find the volume in cubic inches of that part of the boiler below the surface of the water $M\,N\,O\,P$, since the contents of a gallon is given in cubic inches, and from it subtract the volume of the tubes in cubic inches.

This may be done by first finding the *total* area of one end of the boiler in square inches, from it subtracting the area of the segment $M\,N\,S$, and the areas of the ends of the tubes in square inches, and then by multiplying the result by the length of the boiler *in inches*.

Total area of one end $= 54^2 \times .7854 = 2{,}290.23$ sq. in.

Area of segment $M\,N\,S$, as found in last example, $= 567.63$ sq. in.

Area of the end of one tube $= 3.25^2 \times .7854 = 8.2958$ sq. in.

Area of the ends of the 60 tubes $= 8.2958 \times 60 = 497.75$ sq. in.

Hence, the area to be subtracted $= 567.63 + 497.75 = 1{,}065.38$ sq. in. Subtracting, $2{,}290.23 - 1{,}065.38 = 1{,}224.85$ sq. in. = net area.

The cubical contents $= 1{,}224.85 \times 16 \times 12 = 235{,}171.2$ cu. in. This, divided by 231, will give the number of gallons; whence, $235{,}171.2 \div 231 = 1{,}018.06$ gallons of water Ans.

EXAMPLES FOR PRACTICE.

1. Find the area in square inches of the convex surface of a bar of iron $4\frac{1}{4}$ inches in diameter, and 8 feet 5 inches long. Ans. 1,348.53 sq. in.

2. Find the area of the entire surface of the above bar.
Ans. 1,376.9 sq. in.

3. What is the area of the entire surface of the hexagonal prism whose base is shown in Fig. 52? Ans. 77.196 sq. in.

4. A multitubular boiler has the following dimensions: diameter, 50 inches; length between heads, 15 feet; number of tubes, 56; outside diameter of tubes, 3 inches; distance of mean water line from top of boiler, 16 inches. (*a*) Compute the steam space in cubic feet. (*b*) Find the number of gallons of water required to fill the boiler up to the mean water line.
Ans. { (*a*) 56.4 cu. ft. (*b*) 800 gallons.

THE PYRAMID AND CONE.

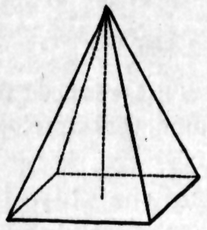

FIG. 54.

418. A **pyramid** (Fig. 54) is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex.**

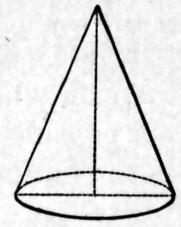

FIG. 55.

419. A **cone** (Fig. 55) is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

420. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

421. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

422. To find the convex area of a pyramid or cone:

Rule 64.—*Multiply the perimeter of the base by one-half the slant height.*

EXAMPLE.—What is the convex area of a pentagonal pyramid, if one side of the base measures 6 inches, and the slant height = 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, has five sides.

$6 \times 5 = 30$ inches, or the perimeter of the base. $30 \times \frac{14}{2} = 210$ sq. in., or the convex area. Ans.

EXAMPLE.—What is the entire area of a right cone whose slant height is 17 inches, and whose base is 8 inches in diameter?

SOLUTION.—The perimeter of the base $= 8 \times 3.1416 = 25.1328$ in.

Convex area $= 25.1328 \times \frac{17}{2} = 213.63$ sq. in.

Area of base $= 8^2 \times .7854 = 50.27$ sq. in.

Entire area $= 263.90$ sq. in. **Ans.**

423. To find the volume of a pyramid or cone:

Rule 65.—*Multiply the area of the base by one-third of the altitude.*

EXAMPLE.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches, and whose altitude is 8 inches?

SOLUTION.—Draw the base as shown in Fig. 56; it will be an equilateral triangle, all of whose sides are 6 inches long.

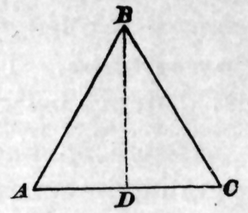

FIG. 56.

Draw a perpendicular BD from the vertex to the base; it will divide the base into two equal parts, since an equilateral triangle is also isosceles, and will be the altitude of the triangle. In order to obtain the area of the base, this altitude must be determined.

In the right-angled triangle BDA, the hypotenuse $BA = 6$ inches, and side $AD = 3$ inches, to find the other side,

$$BD = \sqrt{6^2 - 3^2} = 5.2 \text{ inches, nearly.}$$

Area of the base, or BAC, $= \frac{6 \times 5.2}{2} = 15.6$ sq. in. Hence, the volume $= 15.6 \times \frac{8}{3} = 41.6$ cu. in. Ans.

EXAMPLE.—What is the volume of a cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

SOLUTION.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of a square pyramid whose slant height is 28 inches, and one edge of whose base is $7\frac{1}{2}$ inches long.

Ans. 420 sq. in.

2. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches, and whose altitude is 4 inches?

Ans. 5.2 cu. in.

3. Find the volume of a cone whose altitude is 12 inches, and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base and then its area.

THE FRUSTUM OF A PYRAMID OR CONE.

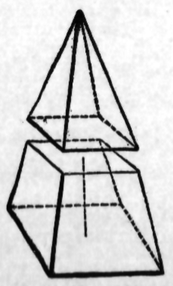

FIG. 57.

424. If a pyramid be cut by a plane, parallel to the base, so as to form two parts, as in Fig. 57, the lower part is called the **frustum** of the pyramid.

If a cone be cut in a similar manner, as in Fig. 58, the lower part is called the **frustum** of the cone.

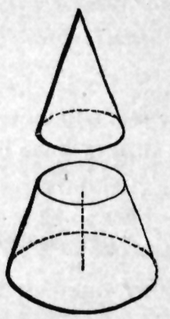

FIG. 58.

425. The upper end of the frustum of a pyramid or cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

426. To find the convex surface of a frustum of a pyramid or cone:

Rule 66.—*Multiply one-half the sum of the perimeters of the two bases by the slant height of the frustum.*

EXAMPLE.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches; find the area of the convex surface.

SOLUTION.—10 in. $\times$ 3 = 30 in., the perimeter of the lower base.
6 in. $\times$ 3 = 18 in., the perimeter of tne upper base.
$\frac{30+18}{2} = 24$ in., or one-half the sum of the perimeters of the two bases. $24 \times 9 = 216$ sq. in., the convex area. Ans.

EXAMPLE.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.—$\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the area of the convex surface.

Area of the upper base = $8^2 \times .7854 = 50.27$ sq. in.
Area of the lower base = $12^2 \times .7854 = 113.1$ sq. in.
The entire area of the frustum = 376.99 + 50.27 + 113.1 = 540.36 sq. in. Ans.

427. To find the volume of the frustum of a pyramid or cone:

Rule 67.—*Add together the areas of the upper and lower bases, and the square root of the product of the two areas; multiply the sum by one-third of the altitude.*

EXAMPLE.—Given, a frustum of a square pyramid (one whose base is a square); each edge of the lower base is 12 inches, each edge of the upper base is 5 inches, and its altitude is 16 inches; what is its volume?

SOLUTION.—Area of upper base = $5 \times 5 = 25$ sq. in.; area of lower base = $12 \times 12 = 144$ sq. in.; the square root of the product of the two areas = $\sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

EXAMPLE.—How many gallons of water will a round tank hold, which is 4 feet in diameter at the top, 5 feet in diameter at the bottom, and 8 feet deep?

SOLUTION.—There are 231 cubic inches in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of the frustum of a cone. The upper diameter = $4 \times 12 = 48$ inches; the lower diameter = $5 \times 12 = 60$ inches, and the depth = $8 \times 12 = 96$ inches. Area of upper base = $48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base = $60^2 \times .7854 = 2{,}827.44$ sq. in.; $\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95$.

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$; $6{,}898.95 \times \frac{96}{3} =$ 220,766.4 cu. in. = contents. Now, since there are 231 cu. in. in one gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gallons, nearly. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of the frustum of a square pyramid, one edge of whose lower base is 15 inches long, one edge of whose upper base is 14 inches long, and whose slant height is one inch.
Ans. 58 sq. in.

2. Find the volume of the above frustum, supposing its altitude to be 3 inches. Ans. 631 cu. in.

3. Find the volume of the frustum of a cone whose altitude is 12 feet and the diameters of whose upper and lower bases are 8 and 10 feet, respectively. Ans. 766.55 cu. ft.

4. If a tank had the dimensions of example 3, how many gallons would it hold? Ans. 5,734.2 gallons, nearly.

THE SPHERE AND CYLINDRICAL RING.

428. A **sphere** (Fig. 59) is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The word **ball,** or **globe,** is generally used instead of sphere.

FIG. 59.

429. To find the area of the surface of a sphere:

Rule 68.—*Square the diameter and multiply the result by 3.1416.*

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION.—Diameter squared $\times$ 3.1416 = $14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75$ sq. in. Ans

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used; a *great circle* of a sphere is the intersection of its surface with a plane passing through its center; for instance, the *great circle* of a sphere 6 in. diameter is a circle of 6 in. diameter. Any number of *great circles* could be described on a given sphere.

430. To find the volume of a sphere:

Rule 69.—*Cube the diameter and multiply the result by .5236.*

EXAMPLE.—What is the weight of a lead ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Diameter cubed $\times$ .5236 $= 12 \times 12 \times 12 \times .5236 = 904.78$ cu. in., or the volume of the ball. The weight, therefore, $= 904.78 \times .41 = 370.96$ pounds. Ans.

431. To find the convex area of a cylindrical ring:

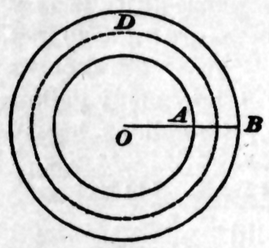

FIG. 60.

A **cylindrical ring** (Fig. 60) is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area:

Rule 70.—*Multiply the circumference of an imaginary cross-section on the line A B, by the length of the center line D.*

EXAMPLE.—If the outside diameter of the ring is 12 inches, and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$, and $10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the inner circle is 4 inches; of the outside circle, 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, $2 \times 3.1416 = 6.2832$ inches, and $6.2832 \times 31.416 =$ 197.4 sq. in., the convex area. Ans.

432. To find the volume of a cylindrical ring:

Rule 71.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D, and whose base is the same as a cross-section of the ring on the line A B, drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on the line A B, by the length of the center line D.*

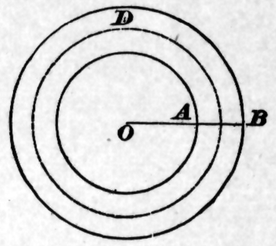

FIG. 61.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches, and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$.

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle = 6 inches; of the inside circle = 4 inches; therefore, the diameter of the cross-section on the line *A B* = 2 inches.

Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cubic inches, the volume. Ans.

EXAMPLES FOR PRACTICE.

1. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in.

2. How many square inches in the surface of the above sphere?
Ans. 2,827.44 sq. in.

3. Required the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches, and the inside diameter $7\frac{1}{2}$ inches. Ans. 107.95 sq. in.

4. Find the cubical contents of the ring in the last example.
Ans. 33.73 cu. in.

5. The surface of a sphere contains 314.16 square inches. What is the volume of the sphere? Ans. 523.6 cu. in

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions and examples contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, and that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

ARITHMETIC.

(SECTION 1.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words:

(*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(6) Represent in figures the following expressions:

(*a*) Seven thousand, six hundred. (*b*) Eighty-one thousand, four hundred, two. (*c*) Five million, four thousand, seven. (*d*) One hundred eight million, ten thousand, one. (*e*) Eighteen million, six. (*f*) Thirty thousand, ten.

(7) What is the sum of $3,290 + 504 + 865,403 + 2,074 + 81 + 7$? Ans. 871,359.

(8) $709 + 8,304,725 + 391 + 100,302 + 300 + 909 =$ what? Ans. 8,407,336.

(9) Find the difference between the following:

(*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

Ans. { (*a*) 47,624. (*b*) 5,338.

(10) (*a*) $70,968 - 32,975 =$? (*b*) $100,000 - 98,735 =$?

Ans. { (*a*) 37,993. (*b*) 1,265.

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(11) The greater of two numbers is 1,004 and their difference is 49; what is their sum? Ans. 1,959.

(12) From 5,962 + 8,471 + 9,023 take 3,874 + 2,039.
Ans. 17,543.

(13) A man willed $125,000 to his wife and two children; to his son he gave $44,675, to his daughter $26,380, and to his wife the remainder. What was his wife's share?
Ans. $53,945.

(14) Find the products of the following:
(*a*) 526,387 × 7; (*b*) 700,298 × 17; (*c*) 217 × 103 × 67.
Ans. (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(15) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800 times.

(16) If a monthly publication contains 24 pages in each issue, how many pages will there be in eight yearly volumes?
Ans. 2,304.

(17) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?
Ans. (*a*) $34,689. (*b*) $37,935.

(18) Solve the following by cancelation:
(*a*) $\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$ (*b*) $\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$
Ans. (*a*) 8. (*b*) 32.

(19) If a mechanic earns $1,500 a year for his labor, and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land, at $133 an acre?
Ans. 7 years.

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week?
Ans. 849 miles.

(21) If the driving wheel of a locomotive is 16 ft. in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance of which is 354 miles, there being 5,280 feet in one mile? Ans. 116,820 rev.

(22) What is the quotient of
(*a*) 589,824 ÷ 576? (*b*) 369,730,620 ÷ 43,911? (*c*) 2,527,525 ÷ 505? (*d*) 4,961,794,302 ÷ 1,234?
Ans. (*a*) 1,024. (*b*) 8,420. (*c*) 5,005. (*d*) 4,020,903.

(23) A man paid $444 for a horse, wagon, and harness. If the horse cost $264 and the wagon $153, how much did the harness cost?
Ans. $27.

(24) What is the product of
(*a*) 1,024 × 576? (*b*) 5,005 × 505? (*c*) 43,911 × 8,420?
Ans. (*a*) 589,824. (*b*) 2,527,525. (*c*) 369,730,620.

(25) If a man receives 30 cents an hour for his wages, how much will he earn in a year, working 10 hours a day and averaging 25 days per month? Ans. $900.

ARITHMETIC.

(SECTION 2.)

EXAMINATION QUESTIONS.

(26) What is a fraction?

(27) What are the terms of a fraction?

(28) What does the denominator show?

(29) What does the numerator show?

(30) How do you find the value of a fraction?

(31) Is $\frac{13}{8}$ a proper or an improper fraction, and why?

(32) Write three mixed numbers.

(33) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(34) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(35) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions. Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(36) What is the value of each of the following: $\frac{13}{2}$, $\frac{17}{4}$, $\frac{69}{16}$, $\frac{16}{8}$, $\frac{67}{64}$? Ans. $6\frac{1}{2}$, $4\frac{1}{4}$, $4\frac{5}{16}$, 2, $1\frac{3}{64}$.

(37) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. (*a*) 112. (*b*) $\frac{3}{16}$. (*c*) $\frac{17}{18}$. (*d*) $4\frac{1}{28}$. (*e*) $3\frac{3}{5}$.

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(38) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8}=?$ Ans. **1.**

(39) $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ Ans. $\frac{15}{16}$.

(40) $42+31\frac{5}{8}+9\frac{7}{16}=?$ Ans. $83\frac{1}{16}$.

(41) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches; and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?

Ans. $190\frac{9}{16}$ sq. in.

(42) Find the value of each of the following:

(*a*) $\frac{7}{\frac{3}{16}}$; (*b*) $\frac{\frac{15}{32}}{\frac{5}{8}}$; (*c*) $\frac{\frac{4+3}{2+6}}{5}$.

Ans. (*a*) $37\frac{1}{3}$. (*b*) $\frac{3}{4}$. (*c*) $\frac{7}{40}$.

(43) The numerator of a fraction is 28, and the value of the fraction $\frac{7}{8}$; what is the denominator? Ans. 32.

(44) What is the difference between (*a*) $\frac{7}{8}$ and $\frac{7}{16}$? (*b*) 13 and $7\frac{7}{16}$? (*c*) $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. (*a*) $\frac{7}{16}$. (*b*) $5\frac{9}{16}$. (*c*) $83\frac{13}{32}$.

(45) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles in another day, and $125\frac{17}{35}$ miles in another day, how far did he travel in the three days? Ans. $289\frac{211}{420}$ miles.

(46) From $573\frac{4}{5}$ tons take $216\frac{5}{8}$ tons. Ans. $357\frac{7}{40}$.

(47) At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $9\frac{1}{4}$ yards of cloth? Ans. $3\frac{15}{32}$ dollars.

(48) Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 by $\frac{7}{8}$ of $\frac{5}{6}$ of 45.

Ans. $109\frac{13}{128}$.

(49) How many times are $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

Ans. 18 times.

(50) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{13}{16}$ pounds.

ARITHMETIC.

(SECTION 3.)

EXAMINATION QUESTIONS.

(51) Write out in words the following numbers: .08, 131, .0001, .000027, .0108, and 93.0101.

(52) How do you place decimals for addition and subtraction?

(53) Give a rule for multiplication of decimals.

(54) Give a rule for division of decimals.

(55) State the difference between a fraction and a decimal.

(56) State how to reduce a fraction to a decimal.

(57) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. .5. .875. .15625. .65. .125.

(58) Solve the following: (*a*) $\frac{32.5+.29+1.5}{4.7+9}$;

(*b*) $\frac{1.283\times\overline{8+5}}{2.63}$; (*c*) $\frac{\overline{589+27}\times\overline{163-8}}{25+39}$;

(*d*) $\frac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}$.

Ans. (*a*) 2.5029. (*b*) 6.3418. (*c*) 1,491.875 (*d*) 8.1139.

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(59) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

(60) What decimal part of a foot is $\frac{3}{16}$ of an inch?
Ans. .015625.

(61) A cubic inch of water weighs .03617 of a pound. What is the weight of a body of water whose volume is 1,500 cubic inches? Ans. 54.255 lb.

(62) If by selling a carload of coal for \$82.50, at a profit of \$1.65 per ton, I make enough to pay for 72.6 ft. of fencing at \$.50 a foot, how many tons of coal were in the car?
Ans. 22 tons.

(63) Divide 17,892 by 231, and carry the result to four decimal places. Ans. 77.4545+.

(64) Find the value of the following expression when the result is carried to three decimal places:

$$\frac{74.26 \times 24 \times 3.1416 \times 19 \times 19 \times 350}{33{,}000 \times 12 \times 4} = ?$$ Ans. 446.619−.

(65) Express (*a*) .7928 in 64ths; (*b*) .1416 in 32ds; (*c*) .47915 in 16ths.
Ans. (*a*) $\frac{51}{64}$. (*b*) $\frac{5}{32}$. (*c*) $\frac{8}{16}$.

(66) Work out the following examples:

(*a*) 709.63 − .8514; (*b*) 81.963 − 1.7; (*c*) 18 − .18; (*d*) 1 − .001; (*e*) 872.1 − (.8721 + .008); (*f*) (5.028 + .0073) − (6.704 − 2.38).
Ans. (*a*) 708.7786. (*b*) 80.263. (*c*) 17.82. (*d*) .999. (*e*) 871.2199. (*f*) .7113.

(67) Work out the following:

(*a*) $\frac{7}{8}$ − .807; (*b*) .875 − $\frac{3}{8}$; (*c*) ($\frac{5}{32}$ + .435) − ($\frac{21}{100}$ − .07); (*d*) what is the difference between the sum of 33-millionths and 17-thousandths, and the sum of 53-hundredths and 274-thousandths?
Ans. (*a*) .068. (*b*) .5. (*c*) .45125. (*d*) .786967.

(68) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(69) 927.416 + 8.274 + 372.6 + 62.07938 = ?
Ans. 1,370.36938

(70) Add 17-thousandths, 2-tenths, and 47-millionths.
Ans. .217047

(71) Find the products of the following expressions:

(*a*) .013 × .107; (*b*) 203 × 2.03 × .203; (*c*) 2.7 × 31.85 × (3.16 − .316); (*d*) (107.8 + 6.541 − 31.96) × 1.742.
Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702

(72) Solve the following:

(*a*) ($\frac{7}{16}$ − .13) × $\overline{.625 + \frac{5}{8}}$; (*b*) ($\frac{19}{32}$ × .21) − (.02 × $\frac{3}{16}$); (*c*) ($\frac{13}{4}$ + .013 − 2.17) × $\overline{13\frac{1}{4} - 7\frac{5}{16}}$.
Ans. (*a*) .384375. (*b*) .1209375. (*c*) 6.4896875.

(73) Solve the following:

(*a*) .875 ÷ $\frac{1}{2}$; (*b*) $\frac{7}{8}$ ÷ .5; (*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125}$.
Ans. (*a*) 1.75. (*b*) 1.75. (*c*) .5.

(74) Find the value of the following expression:

$$\frac{\frac{1.25 \times 20 \times 3}{87 + (11 \times 8)}}{459 + 32}.$$
Ans. $210\frac{3}{7}$.

(75) From 1 plus .001 take .01 plus .000001.
Ans. .990999

ARITHMETIC.

(SECTION 4.)

EXAMINATION QUESTIONS.

(76) What is 25 per cent. of 8,428 lb.? Ans. 2,107 lb.

(77) What is 1 per cent. of $100? Ans. $1.

(78) What is ½ per cent. of $35,000? Ans. $175.

(79) What per cent. of 50 is 2? Ans. 4%.

(80) What per cent. of 10 is 10? Ans. 100%.

(81) Solve the following:

(*a*) Base = $2,522 and percentage = $176.54. What is the rate? (*b*) Percentage = 16.96 and rate = 8 per cent. What is the base? (*c*) Amount = 216.7025 and base = 213.5. What is the rate? (*d*) Difference = 201.825 and base = 207. What is the rate?

Ans. (*a*) 7%. (*b*) 212. (*c*) 1½%. (*d*) 2½%.

(82) A farmer gained 15% on his farm by selling it for $5,500. What did it cost him? Ans. $4,782.61.

(83) A man receives a salary of $950. He pays 24% of it for board, 12½% of it for clothing, and 17% of it for other expenses. How much does he save in a year? Ans. $441.75.

(84) If 37½ per cent. of a number is 961.38, what is the number? Ans. 2,563.68.

(85) A man owns ¾ of a property. 30% of his share is worth $1,125. What is the whole property worth? Ans. $5,000.

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(86) What sum diminished by 35% of itself equals $4,810?
Ans. $7,400.

(87) A merchant's sales amounted to $197.55 on Monday, and this sum was 12½% of his sales for the week. How much were his sales for the week? Ans. $1,580.40.

(88) The distance between two stations on a certain railroad is 16.5 miles, which is 12½% of the entire length of the road. What is the length of the road? Ans. 132 miles.

(89) After paying 60% of my debts, I find that I still owe $35. What was my whole indebtedness? Ans. $87.50.

(90) Reduce 28 rd. 4 yd. 2 ft. 10 in. to inches.
Ans. 5,722 in.

(91) Reduce 5,722 in. to higher denominations.
Ans. 28 rd. 4 yd. 2 ft. 10 in.

(92) How many seconds in 5 weeks and 3.5 days?
Ans. 3,326,400 sec.

(93) How many pounds, ounces, pennyweights, and grains are contained in 13,750 gr.?
Ans. 2 lb. 4 oz. 12 pwt. 22 gr.

(94) Reduce 4,763,254 links to miles.
Ans. 595 mi. 32 ch. 54 li.

(95) Reduce 764,325 cu. in. to cu. yd.
Ans. 16 cu. yd. 10 cu. ft. 549 cu. in.

(96) What is the sum of 2 rd. 2 yd. 2 ft. 3 in.; 4 yd. 1 ft. 9 in.; 2 ft. 7 in.? Ans. 3 rd. 2 yd. 2 ft. 1 in.

(97) What is the sum of 3 gal. 3 qt. 1 pt. 3 gi.; 6 gal. 1 pt. 2 gi.; 4 gal. 1 gi.; 8 qt. 5 pt.? Ans. 16 gal. 3 qt. 2 gi.

(98) What is the sum of 240 gr. 125 pwt. 50 oz. and 3 lb.?
Ans. 7 lb. 8 oz. 15 pwt.

(99) What is the sum of 11° 16′ 12″; 13° 19′ 30″; 20° 25″; 26′ 29″; 10° 17′ 11″? Ans. 55° 19′ 47″.

(100) What is the sum of 130 rd. 5 yd. 1 ft. 6 in.; 215 rd. 2 ft. 8 in.; 304 rd. 4 yd. 11 in.? Ans. 2 mi. 10 rd. 5 yd. 7 in.

(101) What is the sum of 21 A. 67 sq. ch. 3 sq. rd. 21 sq. li.; 28 A. 78 sq. ch. 2 sq. rd. 23 sq. li.; 47 A. 6 sq. ch. 2 sq. rd. 18 sq. li.; 56 A. 59 sq. ch. 2 sq. rd. 16 sq. li.; 25 A. 38 sq. ch. 3 sq. rd. 23 sq. li.; 46 A. 75 sq. ch. 2 sq. rd. 21 sq. li.?
Ans. 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li.

(102) From 20 rd. 2 yd. 2 ft. 9 in. take 300 ft.
Ans. 2 rd. 1 yd. 2 ft. 9 in.

(103) From a farm containing 114 A. 80 sq. rd. 25 sq. yd., 75 A. 70 sq. rd. 30 sq. yd. are sold. How much remains?
Ans. 39 A. 9 sq. rd. 25¼ sq. yd.

(104) From a hogshead of molasses, 10 gal. 2 qt. 1 pt. are sold at one time, and 26 gal. 3 qt. at another time. How much remains? Ans. 25 gal. 2 qt. 1 pt.

(105) If a person were born June 19, 1850, how old would he be August 3, 1892? Ans. 42 yr. 1 mo. 14 da.

(106) A note was given August 5, 1890, and was paid June 3, 1892. What length of time did it run?
Ans. 1 yr. 9 mo. 28 da.

(107) What length of time elapsed from 16 min. past 10 o'clock A. M., July 4, 1883, to 22 min. before 8 o'clock P. M., Dec. 12, 1888? Ans. 5 yr. 5 mo. 8 da. 9 hr. 22 min.

(108) If 1 iron rail is 17 ft. 3 in. long, how long would 51 rails be, if placed end to end? Ans. 53 rd. 1½ yd. 9 in.

(109) Multiply 3 qt. 1 pt. 3 gi. by 4.7.
Ans. 4 gal. 2 qt. 1.7 gi.

(110) Multiply 3 lb. 10 oz. 13 pwt. 12 gr. by 1.5.
Ans. 5 lb. 10 oz. 6 gr.

(111) How many bushels of apples are contained in 9 bbl., if each barrel contains 2 bu. 3 pk. 6 qt.?
Ans. 26 bu. 1 pk. 6 qt.

(112) Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.
Ans. 13 T. 3 cwt. 67.85 lb.

(113) Divide 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft. by 7.
Ans. 51 A. 31 sq. rd. 8 sq. ft.

(114) Divide 282 bu. 3 pk. 1 qt. 1 pt. by 12.
Ans. 23 bu. 2 pk. 2 qt. ¼ pt.

(115) How many iron rails, each 30 ft. long, are required to lay a railroad track 23 miles long? Ans. 8,096 rails.

(116) How many boxes, each holding 1 bu. 1 pk. and 7 qt., can be filled from 356 bu. 3 pk. and 5 qt. of cranberries?

Ans. 243 boxes.

(117) If 16 square miles are equally divided into 62 farms, how much land will each contain?

Ans. 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80 + sq. in.

ARITHMETIC.

(SECTION 5.)

EXAMINATION QUESTIONS.

(118) What is the square of 108? Ans. 11,664.

(119) Find the fifth power of 9. Ans. 59,049.

(120) What is the value of $.0133^3$? Ans. .000002352637.

(121) In what respect does evolution differ from involution?

(122) Extract the square root of 90. Ans. 9.4868+.

(123) Find the value of $(3\frac{3}{4})^3$. Ans. $52\frac{47}{64}$, or 52.734375.

(124) What is the cube root of 92,416? Ans. 45.212—.

(125) Find the value of $\sqrt{502,681}$. Ans. 709.

(126) What is the value of $\sqrt[3]{\frac{27}{64}}$? Ans. $\frac{3}{4}$.

(127) From the cube of 4 take the cube root of 8.

Ans. 62.

(128) What is the value of $\sqrt[3]{\frac{3}{8}}$? Ans. .72112+.

(129) Extract the square root of .3364. Ans. .58.

(130) Find the square root of 3.1416. Ans. 1.7725—.

(131) What number multiplied by itself equals 114.9184?

Ans. 10.72.

(132) Extract the square root of 3,486,784.

Ans. 1,867.3—.

(133) Find the square root of .00041209. Ans. .0203

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ARITHMETIC.

(SECTION 6.)

EXAMINATION QUESTIONS.

Find the value of x in the following:

(143) $11.7 : 13 :: 20 : x$. Ans. 22.22+.

(144) (*a*) $20 + 7 : 10 + 8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$. Ans. (*a*) 2. (*b*) 277.7+.

(145) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$. Ans. (*a*) $x = 12$; (*b*) $x = 12$; (*c*) $x = 20$; (*d*) $x = 180$; (*e*) $x = 40$.

(146) $x : 5 :: 27 : 12.5$. Ans. $10\frac{4}{5}$.

(147) $45 : 60 :: x : 24$. Ans. 18.

(148) $x : 35 :: 4 : 7$. Ans. 20.

(149) $9 : x :: 6 : 24$. Ans. 36.

(150) $\sqrt[3]{1,000} : \sqrt[3]{1,331} = 27 : x$. Ans. 29.7.

(151) $64 : 81 = 21^2 : x^2$. Ans. 23.625.

(152) $7 + 8 : 7 = 30 : x$. Ans. 14.

(153) A man whose steps measure 2 ft. 5 in. takes 2,480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance? Ans. 2,320 steps.

(154) If a horse travels 12 mi. in 1 hr. 36 min., how far will he travel at the same rate in 15 hr.? Ans. 112.5 mi.

(155) If a column of mercury 27.63 in. high weighs .76 of a pound, what will be the weight of a column of mercury having the same diameter, 29.4 inches high? Ans. .808+lb.

(156) If 2 gal. 3 qt. 1 pt. of water will last a man 5 da., how long will 5 gal. 3 qt. last him, if he drinks at the same rate? Ans. 10 da.

(157) Heat from a burning body varies inversely as the square of the distance from it. If a thermometer held 6 ft.

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from a stove shows a rise in temperature of 24 degrees, how many degrees rise in temperature would it indicate if held 12 ft. from the stove? Ans. 6°.

(158) If a pile of wood 12 ft. long, 4 ft. wide, and 3 ft. high is worth $12, what is the value of a pile of wood 15 ft. long, 5 ft. wide, and 6 ft. high? Ans. $37.50.

(159) If 100 gal. of water run over a dam in 2 hr., how many gallons will run over the dam in 14 hr. 28 min.?
Ans. $723\frac{1}{3}$ gal.

(160) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 510 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep?
Ans. $309\frac{9}{14}$ bbl.

(161) If a railway train runs 444 mi. in 8 hr. 40 min., in what time can it run 1,060 mi. at the same rate of speed?
Ans. 20 hr. 41.44 min.

(162) If sound travels at the rate of 6,160 ft. in $5\frac{1}{2}$ sec., how far does it travel in 1 min.? Ans. 67,200 ft.

(163) If 5 men by working 8 hours a day can do a certain amount of work, how many men by working 10 hours a day can do the same work? Ans. 4 men.

(164) If a man travel 540 miles in 20 days of 10 hours each, how many hours a day must he travel to cover 630 miles in 25 days? Ans. $9\frac{1}{3}$ hr.

(165) Referring to example 4, Art. **349,** what is the horsepower of an engine whose cylinder is 30 in. in diameter, piston speed 660 ft. per min., and mean effective pressure 42 lb. per sq. in.? Ans. 594 horsepower.

(166) The weight of a cubic inch of cast iron is .261 pound. Referring to Art. **345,** what is the weight of a solid cast-iron cylinder whose diameter is 12 inches and length is 60 inches? Ans. 1,771.11 lb.

(167) Referring to Art. **348,** what is the centrifugal force of a 40-pound body revolving in a circle having a radius of 10 inches, at a speed of 18 feet per second?
Ans. 484.7 lb.

MENSURATION AND USE OF LETTERS IN FORMULAS.

(SEE ARTS. 340–432.)

EXAMINATION QUESTIONS.

$A = 5$ $\quad h = 200$

$B = 10$ $\quad x = 12$

$i = 3.5$ $\quad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(142) $C = \dfrac{D - x}{B + i}$. Ans. $C = 8$.

(143) $Q = \dfrac{A\,h + D}{2\,x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(144) $r = \dfrac{3.246\,B\,h}{\dfrac{A\,x + h}{A\,i - B}}$. Ans. $r = 187.269+$.

(145) $v = \sqrt{\dfrac{A\,D}{i\,B + 1.5}}$. Ans. $v = 4.05+$.

(146) $u = \sqrt[3]{\dfrac{B\,x}{.00018\,h\,(A^2 - x)}}$. Ans. $u = 6.35+$.

(147) $f = \dfrac{10\,(h - D)^2}{\sqrt[3]{D + A}}$. Ans. $f = 12{,}800$.

(148) $g = \dfrac{(B - A)^2 - \sqrt[3]{D + A}}{A^3 - (1 + D)}$. Ans. $g = 5$.

(149) $k = \sqrt{\dfrac{A\,B^2}{\sqrt[3]{A\,h}}}$. Ans. $k = 7.071+$.

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(150) $T = \sqrt{\dfrac{A^2\left[490 + \dfrac{(h\,x)^2}{D^2}\right]}{h + \dfrac{x}{D}(A^2 - B)^2}}$. Ans. $T = 10$.

(151) If one of the angles formed by one straight line meeting another straight line equals 152° 3′, what is the other angle equal to? Ans. 27° 57′.

(152) How many seconds are there in 140° 17′ 10″.
Ans. 505,030″.

(153) Write a definition for a degree, as applied to the measurement of angles.

(154) (*a*) How many degrees are there in 240 minutes? (*b*) How many seconds? Ans. (*a*) 4°. (*b*) 14,400″.

(155) Draw an obtuse angle, a right angle, and an acute angle. State the name of each angle by using letters to designate them.

(156) Draw a rhombus and then draw a rectangle having the same area.

(157) Can a quadrilateral be formed with lines whose lengths are 20 inches, 9 inches, 4 inches, and 7 inches? Give reasons?

(158) A sheet of zinc measures $11\frac{1}{2}$ inches by $2\frac{1}{2}$ feet. How many square inches does it contain? Ans. 345 sq. in.

(159) If the zinc in the last example weighs $5\frac{1}{4}$ pounds, what is its weight per square foot? Ans. 2.19 lb.

(160) How many boards 16 feet long and 5 inches wide would be required to lay a floor measuring 15 × 24 feet?
Ans. 54 boards.

(161) A lot of land is in the shape of a trapezoid. It is 16 rods long, 9 rods wide at the front, and 6 rods wide at the back. The front and back being parallel, what part of an acre does the lot contain? Ans. $\frac{3}{4}$ of an acre.

(162) The accompanying figure shows the floor plan of

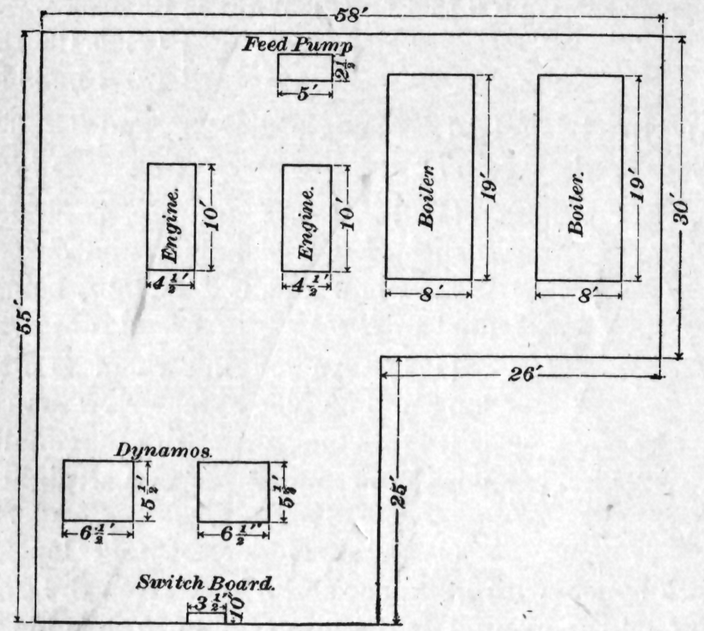

FIG. 1.

an electric-light station. From the dimensions given, calculate the number of square feet of unoccupied floor space.
Ans. 2,059.08 sq. ft.

(163) A sidewalk 10 feet wide extends around a city block 528 feet long and 352 feet wide. Assuming that there is no space between the walk and the buildings, how many square yards does the walk contain? Ans. 2,000 sq. yd.

(164) How many square yards of plastering will be required for the four side walls of a hall 90 feet long, 50 feet wide, and 20 feet high, with four doors $5\frac{1}{2} \times 10$ feet and fourteen windows 5 × 11 feet? There is a baseboard 9 inches high. Ans. 490.72 sq. yd.

(165) A triangle has three equal angles; what is it called?

(166) If a triangle has two equal angles, what kind of a triangle is it?

(167) Can a triangle be formed with three lines whose lengths are 12 inches, 7 inches, and 4 inches? Give reasons for your opinion.

(168) (*a*) What is the altitude of an equilateral triangle whose sides are each 6 feet? (*b*) What is its area?

Ans. { (*a*) 5.196 ft. (*b*) 15.588 sq. ft.

(169) In a triangle $A\,B\,C$, angle $A = 23°$, and $B = 32°\ 32'$; what does angle C equal? Ans. $C = 124°\ 28'$.

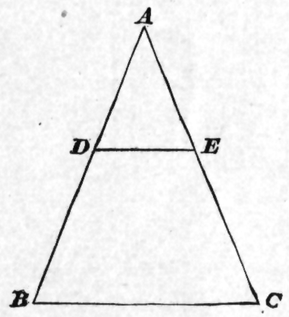

FIG. 2.

(170) In the figure, if $A\ D = 10$ inches, $A\,B = 24$ inches, and $B\ C = 13\frac{1}{2}$ inches, how long is $D\,E$, $D\,E$ being parallel to $B\,C$? Ans. $D\,E = 5.625$ in.

(171) An engine room is 52 feet long and 39 feet wide. How many feet is it from one corner to a diagonally opposite one, measured in a straight line? Ans. 65 ft.

(172) A ladder 24 feet long rests against a house with its upper end 8 feet from the ground. How far on the ground is the lower end of the ladder from the house? Ans. 22 ft. $7\frac{1}{2}+$ in.

(173) (*a*) Show why it is that the area of a triangle equals one-half the product of the base by the altitude. (*b*) Does it make any difference which side is taken as the base?

(174) (*a*) The area of an isosceles triangle is 200 square inches. If its altitude is 20 inches, how long is its base? (*b*) What is the length of one of its equal sides?

Ans. { (*a*) 20 in. (*b*) 22.36 in.

(175) In an equilateral heptagon one of the sides equals 3 inches; what is the length of the perimeter? Ans. 21 in.

(176) The perimeter of a regular decagon is 40 inches; what is the length of the side? Ans. 4 in.

(177) What is one angle of a regular dodecagon equal to? Ans. 150°.

(178) The area of a regular pentagon is 43 square inches. If one side is 5 inches long, what is the perpendicular distance from the center to one side? Ans. 3.44 in.

(179) It is required to make a miter-box, in which to cut molding to fit around an octagon post. At what angle with the side of the box should the saw run? Ans. $67\frac{1}{2}°$.

(180) Calculate the area of the irregular polygon, Fig. 3. The dimensions are to be obtained by measuring. Ans. 1.78 + sq. in.

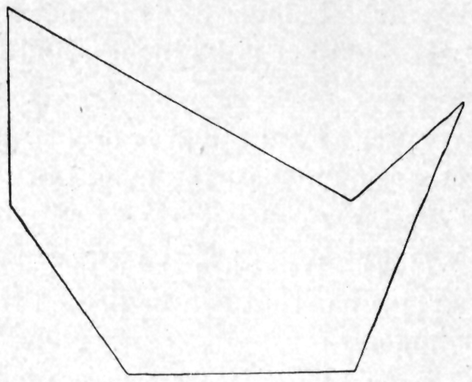
FIG. 3.

(181) An angle inscribed in a circle intercepts one-fourth the circumference. How many degrees are there in the angle? Ans. 45°.

(182) If the distance between two opposite corners of a hexagonal nut is two inches, what is the distance between two opposite sides? Ans. 1.732 + in.

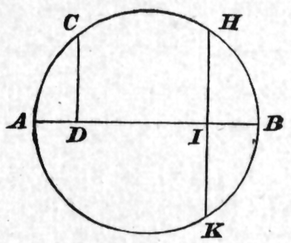

FIG. 4.

(183) In the accompanying figure, if the distance $B\,I$ is 6 inches and $H\,K$ 18 inches, what is the diameter of the circle? Ans. 19.5 in.

(184) In the same figure, if the diameter $A\ B = 32\frac{1}{2}$ feet, and the distance $I\,B = 8$ feet, what is the length of the chord $H\,K$? Ans. 28 ft.

(185) The trunk of a tree measures 7.854 feet around it; what is its diameter? Ans. $2\frac{1}{2}$ ft.

(186) How many revolutions will a 72-inch locomotive driver make in going one mile? Ans. 280.112 revolutions.

(187) A pipe has an internal diameter of 6.06 inches; what is the area of a circle having this diameter?

Ans. 28.8427 sq. in.

(188) The area of a circle having a diameter corresponding to the internal diameter of a certain pipe is 113.0976 square inches. What is the outside diameter, the pipe being $\frac{3}{4}$ inch thick? Ans. $13\frac{1}{2}$ inches.

(189) How long must the arc of a circle be to contain 12°, supposing the radius of the circle to be 6 inches?

Ans. 1.25664 in.

(190) Calculate the area of a flat circular ring whose outside diameter is 22 inches, and whose inside diameter is 21 inches. Ans. 33.7722 sq. in.

(191) Find the area of a segment of a circle whose diameter is 56 inches, the height of the segment being 5 inches.

Ans. 108.5 sq. in.

(192) What are the dimensions of the end of the largest square bar that can be planed from an iron bar 2 inches in diameter? Ans. 1.4142 in. square.

(193) What is the area of the sector of a circle 15 inches in diameter, the angle between the two radii forming the sector being 12½°? Ans. 6.1359 sq. in.

(194) (*a*) What would be the length of the side of a square metal plate having an area of 103.8691 square inches? (*b*) What would be the diameter of a round plate having this area? (*c*) How much shorter is the circumference of the round plate than the perimeter of the square plate?

Ans. { (*a*) 10.1916 in. (*b*) 11½ in. (*c*) 4.638 in.

(195) A plate 46 inches long is to be rolled into a shell; what will be the diameter of the shell if 5 inches are allowed for lap? Ans. 13.05 in.

(196) What is the convex area in square feet of a section of a smokestack 26 inches in diameter and 10½ feet long?

Ans. 71.4714 sq. ft.

(197) Find the area in square feet of the entire surface of a hexagonal column 12 feet long, each edge of the ends of the column being 4 inches long? Ans. 24.5774 sq. ft.

(198) Find the cubical contents of the above column in cubic inches. Ans. 5,985.9648 cu. in

(199) Compute the weight per foot of an iron boiler tube 4 inches outside diameter and 3.73 inches inside diameter, the weight of the iron being taken at .28 pound per cubic inch. Ans. 5½ lb.

(200) The dimensions of a return-tubular boiler are as follows: Diameter, 60 inches; length between heads, 16 feet; outside diameter of tubes, 3½ inches; number of tubes, 64; distance of mean water-line from top of boiler, 18 inches. (*a*) Compute the steam space of the boiler in cubic feet. (*b*) Determine the number of gallons of water that will be required to fill the boiler up to the mean water level?

Ans. { (*a*) 79.2 cu. ft. (*b*) 1,246 gal., nearly.

(201) The cylinders of a compound engine are 19 and 31 inches in diameter, and the stroke is 24 inches; if the clearance at *each* end of the small cylinder is 14% of the *stroke*, and in the large cylinder 8% of the *stroke*, (*a*) what is the total volume in cubic feet of the steam in the small cylinder during one stroke? (*b*) In the large cylinder? (*c*) What is the ratio between the two?

Ans. { (*a*) 4.489 cu. ft. (*b*) 11.321 cu. ft. (*c*) Ratio = 2.522 : 1.

(202) Find the volume of a triangular pyramid 10 inches high; the base is an equilateral triangle, and one edge measures 10 inches. Ans. 144.336 cu. in.

(203) The slant height of a square pyramid is 25 inches, and one edge of its base is 16 inches. Find its altitude by the principle of the right-angled triangle. Ans. 23.6854 in.

(204) The length of the circumference of the base of a cone is 18.8496 inches, and its slant height is 10 inches. Find the area of the entire surface of the cone.

Ans. 122.5224 sq. in.

(205) If the altitude of the above cone were 9 inches, what would be its volume? Ans. 84.8232 cu. in.

(206) A square vat is 11 feet deep, 15 feet square at the top, and 12 feet square at the bottom. How many gallons will it hold? Ans. 15,058.29 gal.

(207) How many pails of water would be required to fill the vat, the pail having the following dimensions: Depth, 11 inches; diameter at the top, 12 inches; diameter at the bottom, 9 inches? Ans. 3,627.28.

(208) Required the area of the convex surface of the frustum of a hexagonal pyramid whose slant height is 32 inches, the perimeter of the lower base measuring 48 inches, and of the upper base 36 inches. Ans. 1,344 sq. in.

(209) It is desired to find the number of gallons that a certain tank will hold. By measuring with a tape measure, it is found to be 190 inches in circumference at the bottom and 170½ inches in circumference at the top. The depth of the tank is 7 feet, and the thickness of the sides 1¼ inches. Make the calculation in round numbers. Ans. 861 gal.

NOTE.—To obtain the area of the upper and lower bases, first find the outside diameters and then deduct the thickness of the walls, or $2 \times 1\frac{1}{4}$ inches.

(210) Find (*a*) the area of the surface, and (*b*) the cubical contents of a ball 22½ inches in diameter.

Ans. { (*a*) 1,590.435 sq. in.
(*b*) 5,964.1313 cu. in.

(211) What is the volume of a ball whose surface has an area of 201.0624 square inches? Ans. 268.0832 cu. in.

(212) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight? Take the weight of 1 cubic inch of cast iron as .261 pound.

Ans. Weight = 21 lb.

(213) The volume of a certain cylindrical ring is 144.349 cu. in., its length before bending (see line *D* in Fig. 61, Art. **432**) was 20.42 inches; what is the area of its surface?

Ans. 192.45 sq. in.

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED BETWEEN

PAGES 151 AND 174 OF THIS VOLUME.

It will be noticed that the Key is divided into sections which correspond to the sections contained in the Examination Questions. The answers and solutions are so numbered as to be similar to the numbers before the questions to which they refer.

To be of the greatest benefit, the Keys should be used sparingly. They should be used much in the same manner as a pupil would go to a teacher for instruction with regard to answering some example he was unable to solve. If used in this manner, the Keys will be of great help and assistance to the student, and will be a source of encouragement to him in studying the various papers composing the Course.

ARITHMETIC.

(SECTIONS 1–3.)

(QUESTIONS 1–75.)

---

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four,

9,006,042 = Nine million, six thousand and forty-two.

850,317,002 = Eight hundred fifty million, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million, four thousand and seven = 5,004,007.

One hundred and eight million, ten thousand and one = 108,010,001.

Eighteen million and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column, and reserve 1 ten for the column

```
  3290
   504
865403
  2074
    81
     7
------
871359
```

Ans.

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of tens. $1+8+7+9=25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2=13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3=11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1+6=7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure 8 in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0=19$; write the nine and reserve the 1. $1+8+7+0+0+9=25$; write the 5 and reserve the 2. $2+0+4+5+2=13$; write the 3 and reserve the 1. $1+2+5+3=11$; write the 1 and reserve 1. $1+6=7$; write the 7. Bring down the 8 to its place in the sum.

(8)

```
    709
8304725
    391
 100302
    300
    909
-------
8407336   Ans.
```

(9) (*a*) In subtracting whole numbers, place the subtrahend or smaller number under the minuend or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. As 1 *ten* = 10 *units*, we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so

only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

```
50962
 3338
-----
47624   Ans.
```

(*b*)

```
15339
10001
-----
 5338   Ans.
```

(10) (*a*)

```
70968
32975
-----
37993   Ans.
```

(*b*)

```
100000
 98735
------
  1265   Ans.
```

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction we have

```
1004
----
  49
```

in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

```
1004  larger
 955  smaller
----
1959  sum.   Ans.
```

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004)

and the difference (49); or, 1,004 − 49 = 955, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959 sum. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

5962	3874
8471	2039
9023	5913 *sum.*
23456 *sum.*	

Subtracting the smaller number (5,913) from the greater (23,456) we have

23456
5913
17543 *difference.* Ans.

(13) $44675 = amount willed to his son.
26380 = amount willed to his daughter.
$71055 = amount willed to his two children.
$125000 = amount willed to his wife and two children.
71055 = amount willed to his two children.
$53945 = amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)
526387
7
3684709 Ans.

7 × 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 tens + 4 tens reserved = 60 tens or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve

the 2 thousands. 7 × 6 thousands = 42 thousands; 42 + 2 thousands reserved = 44 thousands or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve the 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

(*b*)
700298
17
4902086
700298
11905066 Ans.

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700298 = 700298, and write 700298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*) 217 Multiply any two of the numbers together
103 and multiply their product by the third
651 number.
2170
22351
67
156457
134106
1497517 Ans.

(15) If your watch ticks every second, then to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

60 seconds = 1 minute.
60 minutes = 1 hour.
3600 seconds = 1 hour.
24 hours = 1 day.
14400
7200
86400 seconds = 1 day.
7 days = 1 week.
604800 seconds in 1 week or the number of times that
Ans. your watch ticks in 1 week.

(16) If a monthly publication contains 24 pages, a yearly
24 volume will contain 12×24 or 288 pages, since
12 there are 12 months in one year; and eight
288 yearly volumes will contain 8×288, or 2,304
8 pages.
2304 Ans.

(17) If an engine and boiler are worth $3,246, and the building is worth 3 times as much, plus $1,200, then the building is worth

$3246
3
9738
plus 1200
$10938 = value of building.

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$10938
2
21876
plus 1875
$23751 = value of tools.

Value of building = $10938
Value of tools = 23751
$34689 = value of the building and tools. (*a*) Ans.

Value of engine and boiler = $ 3246
Value of building and tools = 34689
$37935 = value of the whole plant. (*b*) Ans.

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the *dividend* and 15 in the *divisor* are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{96 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

The 5 and 15 are *not* to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and 15 *never* existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the *dividend* and 96 in the *divisor* are *divisible* by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 96 and write the 8 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times 1}{\underset{8}{\cancel{96}} \times 3 \times 7 \times 6} =$$

The 72 and 96 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 *never* existed. Thus,

$$\frac{6 \times 48 \times 28 \times 1}{8 \times 3 \times 7 \times 6} =$$

Again, 28 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{6 \times 48 \times \overset{4}{\cancel{28}} \times 1}{8 \times 3 \times \underset{1}{\cancel{7}} \times 6} =$$

The 28 and 7 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 *never* existed. Thus,

$$\frac{6 \times 48 \times 4 \times 1}{8 \times 3 \times 1 \times 6} =$$

Again, 48 in the *dividend* and 6 in the *divisor* are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, cross off the 6 and write the 1 *under* it. Thus,

$$\frac{6 \times \overset{8}{\cancel{48}} \times 4 \times 1}{8 \times 3 \times 1 \times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 *never* existed. Thus,

$$\frac{6 \times 8 \times 4 \times 1}{8 \times 3 \times 1 \times 1} =$$

Again, 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, cross off the 3 and write the 1 *under* it. Thus,

$$\frac{\overset{2}{\cancel{6}} \times 8 \times 4 \times 1}{8 \times \underset{1}{\cancel{3}} \times 1 \times 1} =$$

The 6 and 3 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{2 \times 8 \times 4 \times 1}{8 \times 1 \times 1 \times 1} =$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2 \times \overset{1}{\cancel{8}} \times 4 \times 1}{\underset{1}{\cancel{8}} \times 1 \times 1 \times 1} = \frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1}.$$

Since there are *no two remaining numbers* (one in the dividend and one in the divisor) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the *dividend* together, and divide their *product* by the *product* of all the *uncanceled* numbers in the divisor. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the *dividend* equals $2 \times 1 \times 4 \times 1 = 8$; the product of all the *uncanceled* numbers in the *divisor* equals $1 \times 1 \times 1 \times 1 = 1$.

Hence, $$\frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1} = \frac{8}{1} = 8.$$ Ans.

Or, $$\frac{\overset{2}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{8}{\cancel{96}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} = \frac{8}{1} = 8.$$ Ans.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the *dividend* and 70 in the *divisor* are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 *never* existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the *dividend* and 20 in the *divisor* are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are *not* to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 1} =$$

Again, 16 in the *dividend* and 24 in the *divisor* are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also cross off the 24 and write the 3 *under* it. Thus,

$$\frac{4 \times 60 \times 5 \times \overset{2}{\cancel{16}} \times 14}{7 \times 50 \times \underset{3}{\cancel{24}} \times 1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 2 \times 14}{7 \times 50 \times 3 \times 1} =$$

Again, 60 in the *dividend* and 50 in the *divisor* are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{4 \times \overset{6}{\cancel{60}} \times 5 \times 2 \times 14}{7 \times \underset{5}{\cancel{50}} \times 3 \times 1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 14}{7 \times 5 \times 3 \times 1} =$$

The 14 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times 5 \times 3 \times 1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 2}{1 \times 5 \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{4\times 6\times \overset{1}{\cancel{5}}\times 2\times 2}{1\times \underset{1}{\cancel{5}}\times 3\times 1}=$$

The 5 in the *dividend* and 5 in the *divisor* are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 *never* existed. Thus,

$$\frac{4\times 6\times 1\times 2\times 2}{1\times 1\times 3\times 1}=$$

The 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{4\times \overset{2}{\cancel{6}}\times 1\times 2\times 2}{1\times 1\times \underset{1}{\cancel{3}}\times 1}=$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\frac{32}{1}=32.\quad \text{Ans.}$$

Hence, $\dfrac{\overset{4}{\cancel{80}}\times \overset{2}{\cancel{60}}\times \overset{1}{\cancel{50}}\times \overset{2}{\cancel{16}}\times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{70}}\times \underset{1}{\cancel{50}}\times \underset{1}{\cancel{24}}\times \underset{1}{\cancel{20}}}=\dfrac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\dfrac{32}{1}=32.$ Ans.

(19) 28 acres of land at \$133 an acre would cost $28 \times \$133 = \$3{,}724$.

$$\begin{array}{r} 28 \\ \hline 1064 \\ 266 \\ \hline \$3724 \end{array}$$

If a mechanic earns \$1,500 a year and his expenses are \$968 per year, then he would save \$1500—\$968, or \$532 per year.

$$\begin{array}{r} 968 \\ \hline \$532 \end{array}$$

If he saves \$532 in 1 year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{r} 532)3724(7 \text{ years.} \quad \text{Ans.} \\ 3724 \\ \hline \end{array}$$

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) — 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{rr} & 365 \\ & 3 \\ \hline & 1095 \\ & 246 \\ \hline \textit{difference} & 849 \text{ miles.} \quad \text{Ans.} \end{array}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in one mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

$$\begin{array}{r} 16)1869120(116820 \text{ rev.} \quad \text{Ans.} \\ 16 \\ \hline 26 \\ 16 \\ \hline 109 \\ 96 \\ \hline 131 \\ 128 \\ \hline 32 \\ 32 \\ \hline 0 \end{array}$$

(22) (*a*) 576)589824(1024 Ans.

576

1382

1152

2304

2304

(*b*) 43911)369730620(8420 Ans.

351288

184426

175644

87822

87822

0

(*c*) 505)2527525(5005 Ans.

2525

2525

2525

(*d*) 1234)4961794302(4020903 Ans.

4936

2579

2468

11143

11106

3702

3702

(23) The harness evidently cost the difference between \$444 and the amount which he paid for the horse and wagon.

Since \$264 + \$153 = \$417, the amount paid for the horse and wagon, \$444 − \$417 = \$27, the cost of the harness.

\$264 + 153 = \$417

\$444 − 417 = \$27 Ans.

(24) (*a*)

1024

576

6144

7168

5120

589824 Ans.

(*b*)

5005

505

25025

250250

2527525 Ans.

(*c*)

43911

8420

878220

175644

351288

369730620 Ans.

(25) Since there are 12 months in a year, the number of days the man works is $25 \times 12 = 300$ days. As he works 10 hours each day, the number of hours that he works in one year is $300 \times 10 = 3{,}000$ hours. Hence, he receives for his work $3{,}000 \times 30 = 90{,}000$ cents, or $90{,}000 \div 100 = \$900$. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.* Ans.

(35) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction* and *add the numerator of the fraction to that product.* This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number.*

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\tfrac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16}; \quad 10\tfrac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many units (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) unit remaining. Hence, $\frac{13}{12} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number. Ans.

$\frac{17}{4} = 4\frac{1}{4}$. Ans. $\frac{69}{16} = 4\frac{5}{16}$. Ans. $\frac{16}{8} = 2$. Ans. $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, *invert the divisor* (or, in other words, turn it upside down) *and proceed as in multiplication.*

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1,808}{448} = \frac{452}{112} =$

$\frac{113}{28}$)113(4$\frac{1}{28}$. Ans.
112
1

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4)+3}{4} = \frac{60+3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8)+3}{8} = \frac{32+3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$\frac{18}{5})18(3\frac{3}{5}$ Ans.
$\frac{15}{3}$

(38) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8} = \frac{1+2+5}{8} = \frac{8}{8} = 1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to sixteenths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms

of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1}{4} \times \frac{4}{4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16} = \frac{4+6+5}{16} = \frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers we have $\frac{10}{16}+\frac{7}{16} = \frac{10+7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

42
31
9
$1\frac{1}{16}$
$83\frac{1}{16}$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4}+50\frac{5}{8}+41+69\frac{3}{16}=?$ $\frac{3}{4}=\frac{3\times4}{4\times4}=\frac{12}{16}$.

$\frac{5}{8}=\frac{5\times2}{8\times2}=\frac{10}{16}$. $\frac{12}{16}+\frac{10}{16}+\frac{3}{16}=\frac{12+10+3}{16}=\frac{25}{16}=1\frac{9}{16}$

The problem now becomes $29+50+41+69+1\frac{9}{16}=?$

$$\begin{array}{rl} 29 & \text{square inches.}\\ 50 & \text{square inches.}\\ 41 & \text{square inches.}\\ 69 & \text{square inches.}\\ 1\frac{9}{16} & \text{square inches.}\\ \hline 190\frac{9}{16} & \text{square inches.} \quad \text{Ans.} \end{array}$$

(42) (*a*) $\frac{\frac{7}{3}}{\frac{}{16}}=7\div\frac{3}{16}=7\times\frac{16}{3}=\frac{7\times16}{3}=\frac{112}{3}=37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\frac{\frac{15}{32}}{\frac{5}{8}}=\frac{15}{32}\div\frac{5}{8}=\frac{15}{32}\times\frac{8}{5}=\frac{\overset{3}{\cancel{15}}\times\overset{}{\cancel{8}}}{\underset{4}{\cancel{32}}\times\cancel{5}}=\frac{3}{4}$. Ans.

(*c*) $\frac{\frac{4+3}{2+6}}{5}=\frac{\frac{7}{8}}{5}=\frac{7}{8\times5}=\frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32}=\frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8}-\frac{7}{16}=?$ When the *denominators* of fractions are *not alike* it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the fractions to fractions having a common denominator.* Then, *subtract the numerators, and place the remainder over the common denominator.*

$\frac{7\times2}{8\times2}=\frac{14}{16}$. $\frac{14}{16}-\frac{7}{16}=\frac{14-7}{16}=\frac{7}{16}$. Ans.

(*b*) $13-7\frac{7}{16}=?$ This problem may be solved in two ways:

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$$\begin{array}{r} 12\frac{16}{16}\\ 7\frac{7}{16}\\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$ and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13=\frac{13}{1}=\frac{13\times16}{1\times16}=\frac{208}{16}$. $7\frac{7}{16}=\frac{(7\times16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}$.

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and

$$\begin{array}{l} \frac{89}{16}=16)89(5\frac{9}{16}\\ \quad\frac{80}{}\\ \quad\frac{9}{}\\ \quad\frac{}{16} \end{array}$$

the same result that was obtained by the first method.

(*c*) $312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this we have $\frac{9}{16}=\frac{9\times2}{16\times2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$ and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32}\\ 229\frac{5}{32}\\ \hline 83\frac{13}{32} \end{array}$$

$83+\frac{13}{32}=83\frac{13}{32}$. Ans.

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles.

Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

	85
	78
	125
	$1\frac{2}{4}\frac{1}{2}\frac{1}{0}$
	$289\frac{2}{4}\frac{1}{2}\frac{1}{0}$

To find the least common denominator, we have

$$\begin{array}{r|l} 5 & 12,\ 5,\ 35 \\ 7 & 12,\ 1,\ 7 \\ \hline & 12,\ 1,\ 1, \end{array} \quad \text{or } 5\times7\times12=420.$$

(46)

$573\frac{4}{5}$ tons.	$\frac{4}{5}=\frac{32}{40}$
$216\frac{5}{8}$ tons.	$\frac{5}{8}=\frac{25}{40}$
difference $357\frac{7}{40}$ tons. Ans.	$\frac{7}{40}$ = *difference.*

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans

(48) Referring to Arts. **114** and **116,**

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of $45=$

$$\frac{\cancel{2}\times\cancel{3}\times7\times19\times\cancel{11}\times7\times5\times\overset{3}{\cancel{45}}}{\cancel{3}\times4\times\cancel{11}\times\underset{4}{\cancel{20}}\times1\times8\times\underset{\cancel{3}}{\cancel{6}}\times1}=\frac{7\times19\times7\times5\times3}{4\times4\times8}=\frac{13,965}{128}=$$

$109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.$ $12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers

to improper fractions. $\frac{845}{\cancel{4}}\times\frac{15}{8}=\frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently

$$\frac{12,675}{32}\div2\frac{1}{2}=\frac{\overset{2,535}{\cancel{12,675}}}{\underset{16}{\cancel{32}}}\times\frac{2}{\cancel{5}}=\frac{2,535}{16}=158\frac{7}{16}$$ pounds. The amount remaining is $211\frac{1}{4}-158\frac{7}{16}=\frac{845}{4}-\frac{2,535}{16}=\frac{3,380}{16}-\frac{2,535}{16}=\frac{845}{16}=52\frac{13}{16}$ pounds. Ans.

(51)

tenths.	hundredths.		
0	8	= *Eight* **hundredths.**	

tenths.	hundredths.	thousandths.	
1	3	1	= *One hundred thirty-one* **thousandths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
0	0	0	1	= *One* **ten-thousandth.**

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
0	0	0	0	2	7	= *Twenty-seven* **millionths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
0	1	0	8	= *One hundred eight* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.
93.0	1	0	1	= Ninety-three, and *one hundred one* ***ten-thousandths***

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, *and* one hundred one ten thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator. Ans.

$\frac{7}{8}$)7.000
.875 Ans.

$\frac{5}{32}$)5.00000(.15625 Ans.
32
180
160
200
192
80
64
160
160

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator (32.5 + .29 + 1.5) is to be divided by the denominator (4.7 + 9). The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

32.5
\+ .29
\+ 1.5

13.7)34.29000(2.5029 Ans.
274
689
685
400
274
1260
1233
27

4.7
\+ 9.0
13.7

Since there are 5 decimal places in the dividend and 1 in the divisor, there are 5 − 1 or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283 \times \overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$\frac{1.283 \times \overline{8+5}}{2.63} = ?$ $\quad \overline{8+5} = 13.$

$$\begin{array}{r} 1.283 \\ \times \quad 13 \\ \hline 3849 \\ 1283 \\ \hline \end{array}$$

$$2.63\overline{)16.679000}\,(6.3418 \quad \text{Ans.}$$

$$\begin{array}{r} 1578 \\ \hline 899 \\ 789 \\ \hline 1100 \\ 1052 \\ \hline 480 \end{array} \qquad \begin{array}{r} 480 \\ 263 \\ \hline 2170 \\ 2104 \\ \hline 66 \end{array}$$

(*c*) $\frac{\overline{589+27} \times \overline{163-8}}{25+39} = ?$

$$\begin{array}{r} 589 \\ + \quad 27 \\ \hline 616 \end{array} \qquad \begin{array}{r} 25 \\ + 39 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 163 \\ - \quad 8 \\ \hline 155 \\ \times 616 \\ \hline 930 \\ 155 \\ 930 \\ \hline \end{array}$$

$$64\overline{)95480.000}\,(1491.875 \quad \text{Ans.}$$

$$\begin{array}{r} 64 \\ \hline 314 \\ 256 \\ \hline 588 \\ 576 \\ \hline 120 \\ 64 \\ \hline 560 \\ 512 \\ \hline 480 \\ 448 \\ \hline 320 \\ 320 \\ \hline \end{array}$$

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\frac{\overline{40.6+7.1} \times (3.029 - 1.874)}{6.27 + 8.53 - 8.01} = ?$

$$\begin{array}{r} 40.6 \\ + \quad 7.1 \\ \hline 47.7 \end{array} \qquad \begin{array}{r} 6.27 \\ + 8.53 \\ \hline 14.80 \\ - \quad 8.01 \\ \hline 6.79 \end{array}$$

$$\begin{array}{r} 3.029 \\ - 1.874 \\ \hline 1.155 \\ \times \quad 47.7 \\ \hline 8085 \\ 8085 \\ 4620 \\ \hline \end{array}$$

$$6.79\overline{)55.093500}\,(8.1139. \quad \text{Ans}$$

$$\begin{array}{r} 5432 \\ \hline 773 \\ 679 \\ \hline 945 \\ 679 \\ \hline 2660 \\ 2037 \\ \hline 6230 \\ 6111 \\ \hline 119 \end{array}$$

6 decimal places in the dividend – 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\cancel{8}_{2}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\tfrac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\cancel{12}_{4}} = \frac{1}{64} \text{ of a foot.}$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, $6 - 0 = 6$ places to be pointed off.

$\frac{1}{64}$) 1.000000 (.015625 Ans.
64
360
320
400
384
160
128
320
320

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be $.03617 \times 1{,}500 =$ 54.255 lb.

.03617 lb.
1500
1808500
3617
54.25500 lb. Ans.

(62) 72.6 feet of fencing at $.50 a foot would cost

$72.6 \times .50$, or $36.30.

.50
$36.300

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

1.65) 36.30 (22 tons. Ans.
330
330
330

(63) 231) 17892.00000 (77.45454, or 77.4545 to four decimal places. Ans.
1617
1722
1617
1050
924
1260
1155
1050
924
1260
1155
1050

(64) $\dfrac{37.13 \quad 2 \quad .0952}{\dfrac{74.26 \times 24 \times 3.1416 \times 19 \times 19 \times 350}{33{,}000 \times 12 \times 4} =}$
$\dfrac{1{,}000 \qquad 2}{}$

$$\frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1{,}000} = \frac{446{,}618.947600}{1{,}000} =$$

446.619 to three decimal places. Ans.

37.13 × .0952 = 3.534776
19 × 19 = 361
361 × 350 = 126350
3.534776 × 126350 = 446618.947600

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

(*a*) 709.6300 − .8514 = 708.7786 Ans.

In the above example we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and, as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths;* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*) 81.963 − 1.700 = 80.263 Ans.

(*c*) 18.00 − .18 = 17.82 Ans.

(*d*) 1.000 − .001 = .999 Ans.

(*e*) 872.1 − (.8721 + .008) = ? In this problem we are to subtract (.8721 + .008) from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

.8721 + .008 = .8801 *sum.*

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

872.1000 − .8801 = 871.2199 Ans.

(*f*) (5.028 + .0073) − (6.704 − 2.38) = ? First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

5.0280 + .0073 = 5.0353 *sum.*

6.704 − 2.380 = 4.324 *difference.*

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324. The sign between the parentheses indicates that the quantities obtained by performing the above operations, are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation we obtain .7113 as the final result.

5.0353 − 4.324 = .7113 Ans.

(67) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$\frac{7}{8}$)7.000 = .875

.875 − .807 = .068 Ans.

Subtracting .807 from .875 the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8})\underline{3.000}$ and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

.3 7 5

	.8 7 5
	.3 7 5
	.5 0 0 Ans.

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 57).

Adding .15625 and .435,	.15625	$\frac{21}{100} = .21$; subtracting,		.21
	.425			.07
sum	.59125		*difference*	.14

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is, *difference*

	.59125
	.14
	.45125 Ans.

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

.0	tenths.	.5	tenths.		
.0	hundredths.	.5	hundredths.		
.0	thousandths.	.2	thousandths.		
.0	ten-thousandths.				
.0	hundred-thousandths.				
.0	millionths.				
.0 1 7		.2 7 4			
.0 1 7 0 3 3	*sum.*	.8 0 4	*sum*		

	.804	*larger sum,*
	.017033	*smaller sum,*
difference	.786967	Ans.

(68) In addition of decimals the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

.1 2 5	
.7	
.0 8 9	
.4 0 0 5	
.9	
.0 0 0 0 2 7	
2.2 1 4 5 2 7	Ans.

(69)

9 2 7.4 1 6	
8.2 7 4	
3 7 2.6	
6 2.0 7 9 3 8	
1 3 7 0.3 6 9 3 8	**Ans.**

(70)

.0	tenths.
.0	hundredths.
.0	thousandths.
.0	ten-thousandths.
.0	hundred-thousandths.
.0	millionths.
.0 1 7	
.2	
.0 0 0 0 4 7	
.2 1 7 0 4 7 = Ans.	

(71) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are $3 + 3$ or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

.1 0 7	
.0 1 3	
3 2 1	
1 0 7	
.0 0 1 3 9 1	Ans.

There are two decimal places in the multiplier and none in the multiplicand; hence, there are $2 + 0$ or two decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are $3 + 2$ or 5 decimal places in the second product.

(*b*)

2 0 3	
2.0 3	
6 0 9	
4 0 6 0	
4 1 2.0 9	
.2 0 3	
1 2 3 6 2 7	
8 2 4 1 8 0	
8 3.6 5 4 2 7	Ans.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

The parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r} 3.160 \\ .316 \\ \hline 2.844 \end{array}$$

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

$$\begin{array}{r} 31.85 \\ 2.7 \\ \hline 22295 \\ 6370 \\ \hline 85.995 \end{array}$$

$$\begin{array}{r} 85.995 \\ 2.844 \\ \hline 343980 \\ 343980 \\ 687960 \\ 171990 \\ \hline 244.569780 \end{array}$$ Ans.

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

$$\begin{array}{r} 107.8 \\ + \quad 6.541 \\ \hline 114.341 \\ - \quad 31.96 \\ \hline 82.381 \\ \times \quad 1.742 \\ \hline 164762 \\ 329524 \\ 576667 \\ 82381 \\ \hline 143.507702 \end{array}$$ Ans.

(72) (*a*) $\left(\frac{7}{16} - .13\right) \times \overline{.625 + \frac{5}{8}} = ?$

First perform the operation indicated by the parenthesis.

$\frac{7}{16} = \frac{7}{16})\,7.0000(.4375$

$$\begin{array}{r} 64 \\ \hline 60 \\ 48 \\ \hline 120 \\ 112 \\ \hline 80 \\ 80 \\ \hline \end{array}$$

We point off four decimal places since we annexed four ciphers.

$$\begin{array}{r} .4375 \\ .13 \\ \hline .3075 \end{array}$$

Subtracting, we obtain

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$\frac{5}{8} = \frac{5}{8})\,5.000$

$.625$

Adding the terms included by the vinculum, we obtain

$$\begin{array}{r} .625 \\ .625 \\ \hline 1.250 \end{array}$$

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

$$\begin{array}{r} .3075 \\ 1.25 \\ \hline 15375 \\ 6150 \\ 3075 \\ \hline .384375 \end{array}$$ Ans.

(*b*) $\left(\frac{19}{32} \times .21\right) - \left(.02 \times \frac{3}{16}\right) = ?$

$.21 = \frac{21}{100}$. $\frac{19}{32} \times \frac{21}{100} = \frac{399}{3200}$. $.02 = \frac{2}{100}$. $\frac{2}{100} \times \frac{3}{16} = \frac{6}{1600} = \frac{3}{800}$.

$\frac{3}{800} = \frac{3}{800} \times \frac{4}{4} = \frac{12}{3200}$. $\frac{399}{3200} - \frac{12}{3200} = \frac{399 - 12}{3200} = \frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$$\frac{387}{3200}\,)\,387.0000000\,(\,.1209375 \quad \text{Ans.}$$

```
3200 ) 387.0000000 ( .1209375   Ans.
       3200
        6700
        6400
         30000
         28800
          12000
           9600
           24000
           22400
            16000
            16000
```

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $\left(\frac{13}{4} + .013 - 2.17\right) \times \overline{13\frac{1}{4} - 7\frac{5}{16}} = ?$

$\frac{13}{4} = \frac{13}{4})\,13.00$ — 3.25

Point off two decimal places, since two ciphers were annexed to the dividend.

```
  3.25
+ .013
 3.263
- 2.17
 1.093
```

$\frac{5}{16}$ reduced to a decimal is .3125, since

```
5
16 ) 5.0000 ( .3125
     48
      20
      16
       40
       32
        80
        80
```

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16} = 7.3125$, and $13\frac{1}{4} = 13.25$, since $\frac{1}{4} = \frac{1}{4})\,1.00$ — $.25$


```
  13.25                 5.9375
-  7.3125             × 1.093
   5.9375              178125
                      534375
                     593750
                     6.4896875   Ans.
```

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75.$ Ans.

Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{7}{4} = 1\frac{3}{4}$. Since $\frac{3}{4} = \frac{3}{4})\,3.00\,(.75$, $1\frac{3}{4} = 1.75$, the same answer as above.

```
4 ) 3.00 ( .75
    28
     20
     20
```

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{8} \times \frac{2}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are 3 − 1, or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1,000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1,000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to sixteenths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the de-

nominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \qquad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\cancel{3}}{\cancel{32}_{2}} \times \frac{\cancel{16}}{\cancel{3}} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(74) $\dfrac{\frac{1.25 \times 20 \times 3}{87 + (11 \times 8)}}{459 + 32} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\dfrac{87 + (11 \times 8)}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The numerator is combined as though it were written $87 + (11 \times 8)$, and its result is

$$\begin{array}{r} 11 \\ \quad 8 \text{ times} \\ \hline 88 \\ +87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1,473}{7} = 210\frac{3}{7}. \quad \text{Ans.}$$

(75) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And 1.001 − .010001 =

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array}$$

Ans.

ARITHMETIC.

(SECTION 4.)

(QUESTIONS 76–117.)

(76) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. = .25 × 8,428 lb. = 2,107 lb. Ans.

(77) Here \$100 is the base and 1% = .01 is the rate. Then, .01 × \$100 = \$1. Ans.

(78) $\frac{1}{2}$% means one-half of one per cent. Since 1% is .01, $\frac{1}{2}$% is .005, for, $2\overline{).010}$ $.005$. And .005 × \$35,000 = \$175. Ans.

(79) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **193,**

rate = percentage ÷ base;
rate = 2 ÷ 50 = .04 or 4%. Ans.

(80) By Art. **193,** rate = percentage ÷ base.*
As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1 = 100%. Hence, 10 is 100% of 10. Ans.

(81) (*a*) Rate = percentage ÷ by base. Art. **193.**
As percentage = \$176.54 and base = \$2,522, we have

rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

$$2522\overline{)176.54}\ .07$$

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

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(*b*) Base = percentage ÷ rate. Art. **192.**
As percentage = 16.96 and rate = 8% = .08, we have

base = 16.96 ÷ .08 = 212. Ans.

.08) 16.96
212

(*c*) Amount is the sum of the base and percentage; hence the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}\%$. Ans.

213.5) 3.2025 (.015 = $1\frac{1}{2}\%$
2135
10675
10675

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825, hence percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 5.175 ÷ 207 = .025 = $.02\frac{1}{2} = 2\frac{1}{2}\%$. Ans.

207) 5.175 (.025
414
1035
1035

(82) In this problem $5,500 is the amount, since it equals what he paid for the farm + what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **197,**

base = amount ÷ (1 + rate); hence,
base = $5,500 ÷ (1 + .15) = $4,782.61. Ans.

1.15) 5500.0000 (4782.61
460
900
805
950
920
300
230
700
690
100
115

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = $5,500, the selling price.

If 115% = $5,500, $1\% = \frac{1}{115}$ of $5,500 = $47.8261, and 100%, or the cost, = 100 × $47.8261 = $4,782.61. Ans.

(83) 24 % of $950 = .24 × 950 = $228
$12\frac{1}{2}\%$ of $950 = .125 × 950 = 118.75
17 % of $950 = .17 × 950 = 161.50
$53\frac{1}{2}\%$ of $950 = $508.25

The total amount of his yearly expenses, then, is $508.25, hence his savings are $950 − $508.25 = $441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence, $100\% - 53\frac{1}{2}\% = 46\frac{1}{2}\%$ = per cent. saved. And $950 × .465 = $441.75 = his yearly savings. Ans.

(84) The percentage is 961.38, and the rate is $37\frac{1}{2}$. By Art. **192,**

Base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

```
.375)961.38000(2563.68
     750
     ----
     2113
     1875
     ----
      2388
      2250
      ----
       1380
       1125
       ----
        2550
        2250
        ----
         3000
         3000
         ----
```

(85) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **192,**

Base = percentage ÷ rate = \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of \$3,750 = \$1,250, and $\frac{4}{4}$ or the entire property, is 4 × \$1,250 = \$5,000. Ans.

(86) Here \$4,810 is the difference and 35% the rate. By Art. **198,**

Base = difference ÷ (1 − rate)
= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

```
.65)4810.00(7400
   455
   ---
    260                1.00
    260                 .35
    ---                ----
      00                .65
```

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(87) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}\%$ of the sales for the entire week; i. e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}\%$, and the required number (or the amount of sales for the week) equals the base. By Art. **192,**

Base = percentage ÷ rate = \$197.55 ÷ .125;

or,

```
.125)197.5500(1580.4  Ans.
   125
   ---
    725
    625
    ---
    1005
    1000
    ----
      500
      500
      ---
```

Therefore, base = \$1,580.40, which also equals the sales for the week.

(88) 16.5 miles = $12\frac{1}{2}\%$ of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}\%$ is the rate, and the entire length will be the base. By Art. **192,**

Base = percentage ÷ rate = $16.5 \div .12\frac{1}{2}$.

```
.125)16.500(132 miles.  Ans.
   125
   ---
    400
    375
    ---
     250
     250
     ---
```

(89) Here we have given the difference, or \$35, and the rate, or 60%, to find the base. We use the rule in Art. **198,**

Base = difference ÷ (1 − rate)

= \$35 ÷ (1 − .60) = \$35 ÷ .40 = \$87.50. **Ans.**

```
.40)35.000(87.5
   320
   ---
   300
   280
   ---
    200
    200
    ---
```

Or, 100% = whole debt; 100% − 60% = 40% = \$35.

If 40% = \$35, then $1\% = \frac{1}{40}$ of $\$35 = \frac{35}{40}$, and $100\% = \frac{35}{40} \times 100 = \$87.50$. Ans.

(90) 28 rd. 4 yd. 2 ft. 10 in. to inches.

```
  5½
 ×
  ---
  154
+   4
  ---
  158 yards
×   3
  ---
  474
+   2
  ---
  476 feet
×  12
  ---
 5712
+  10
  ---
 5722 inches.  Ans.
```

Since there are $5\frac{1}{2}$ yards in one rod, in 28 rods there are $28 \times 5\frac{1}{2}$ or 154 yards; 154 yards plus 4 yards = 158 yards. There are 3 feet in one yard; therefore, in 158 yards there are 3×158 or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in one foot, and in 476 feet there are 12×476 or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(91)

```
12 ) 5722 inches.
 3 ) 476 + 10 inches.
5½ ) 158 + 2 feet.
      28 + 4 yards.
```

Ans. = 28 rd. 4 yd. 2 ft. 10 in.

EXPLANATION.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 ft. and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in one rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches there are 28 rd. 4 yd. 2 ft. 10 in.

(92)

```
   5   weeks 3.5 days.
 × 7
 ---
  35   days in 5 weeks.
+ 3.5
 ----
 38.5 days.
```

Then, we find how many seconds there are in 38.5 days.

```
  38.5 days
×   24 hours in one day.
 ----
  1540
  770
 ----
 924.0 hours in 38.5 days.
×   60 minutes in one hour.
 ----
 55440 minutes in 38.5 days.
×   60 seconds in one minute.
 ------
3326400 seconds in 38.5 days.  Ans.
```

(93) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining.

Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

24) 13750 gr.
20) 572 pwt. + 22 gr.
12) 28 oz. + 12 pwt.
2 lb. + 4 oz.

Ans. = 2 lb. 4 oz. 12 pwt. 22 gr.

(94) 100) 4763254 li.
80) 47632 + 54 li.
595 + 32 ch.

Ans. = 595 mi. 32 ch. 54 li.

EXPLANATION.—There are 100 li. in one chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in one mile; hence, in 47,632 ch. there are as many miles as 80 is contained times in 47,632 ch., or 595 miles and 32 ch. remaining.

Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(95) 1728) 764325 cu. in.
27) 442 + 549 cu. in.
16 cu. yd. + 10 cu. ft.

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

EXPLANATION.—There are 1,728 cu. in. in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in one cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(96) We must arrange the different terms in columns, taking care to have like denominations in the same column.

	rd.	yd.	ft.	in.	
	2	2	2	3	
		4	1	9	
			2	7	
	3	$2\frac{1}{2}$	0	7	
or	3	2	2	1	Ans.

EXPLANATION.—We begin to add at the right-hand column. 7 + 9 + 3 = 19 in.; as 12 in. make one foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) + 2 + 1 + 2 = 6 ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) + 4 + 2 = 8 yd. Since $5\frac{1}{2}$ yd. = 1 rod, 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column, and reserve 1 rd. for the next column; 1 (reserved) + 2 = 3 rd.

Ans. =	3 rd.	$2\frac{1}{2}$ yd.	0 ft.	7 in.
or,	3 rd.	2 yd.	1 ft.	13 in.
or,	3 rd.	2 yd.	2 ft.	1 in. Ans.

(97) We write the compound numbers so that the units of the same denomination shall stand in the same column. Beginning to add with the lowest denomination, we find that the sum of the gills is 1 + 2 + 3 = 6. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) + 5 + 1 + 1 = 8. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) + 8 + 3 = 15. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) + 4 + 6 + 3 = 16 gal. Since we can not reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

(98) Reduce the grains, pennyweights, and ounces to higher denominations.

24) 240 gr. 20) 125 pwt. 12) 50 oz.

10 pwt. 6 oz. 5 pwt. 4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.
3		
4	2	
	6	5
		10
7 lb.	8 oz.	15 pwt. Ans.

(99) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is 11 + 29 + 25 + 30 + 12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107″ there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) + 17 + 26 + 19 + 16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) + 10 + 20 + 13 + 11, or 55 degrees. Since we can not reduce 55 degrees to any higher denominations, we have 55° 19′ 47″ for the answer.

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

(100) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11 + 8 + 6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column

of feet. The sum of the feet is 2 feet (reserved) + 2 + 1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) + 4 + 5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) + 304 + 215 + 130 = 650 rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

	rd.	yd.	ft.	in.
	130	5	1	6
	215	0	2	8
	304	4	0	11
	650	$4\frac{1}{2}$	2	1
mi.				
or, 2	10	5	0	7 Ans.

(101) Since "square links" is the lowest denomination in this problem, we find their sum first, which is 21 + 23 + 16 + 18 + 23 + 21, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is 2 + 3 + 2 + 2 + 2 + 3, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

(102) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$ or $\frac{11}{2}$ is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div \frac{11}{2} = 100 \times \frac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

$$11\overline{)200}\,(\,18\tfrac{2}{11}\text{ rd.}$$

$$\begin{array}{r} 11 \\ \hline 90 \\ 88 \\ \hline 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rods there are $\frac{2}{11} \times \frac{11}{2}$, or one yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2 yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

(103)

A.	sq. rd.	sq. yd.
114	80	25
75	70	30
39	9	$25\frac{1}{4}$ Ans.

EXPLANATION.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We can not subtract 30 from 25, so we take one square rod ($= 30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards leaves $25\frac{1}{4}$ square yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from 114 acres, which leaves 39 acres, which we place under the column of acres.

(104) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal 0 + 1, or 1 pint. We can not reduce 1 pint to any higher denomination, so we place it under the pint column. The number of quarts is 3 + 2, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quart column, and reserve the 1 gallon to add to the column of

gal.	qt.	pt.
10	2	1
26	3	0
37 gal.	1 qt.	1 pt.

gallons. The number of gallons equals 1 (reserved) + 26 + 10, or 37 gallons.

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. and 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt. and 1 qt. = 2 pt.

Beginning with the lowest denomination, 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint. One quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(105) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On August 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19 days can not be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have 7 — 1, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1850 from 1892 leaves 42 years. Therefore, he would be 42 years 1 month and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(106) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 can not be subtracted from 3, so we take a unit from the next higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months + 4 months = 16 months. 7 months from 16 months = 9 months. Since we took 1 year from the years column, we have 1892 — 1, or 1891 remaining. 1890 from 1891 leaves 1 year. Hence, the note ran 1 year 9 months and 28 days. Ans.

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

(107) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date, or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(108) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $= 879$ ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3)879 ft. 9 in.
5.5)293 yd.
53 rd. $1\frac{1}{2}$ yd.

Then, answer = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in.; or 53 rd. 1 yd. 2 ft. 3 in.

(109)

	qt.	pt.	gi.
	3	**1**	3
			4.7
	18.2 qt.	0	.1
or,	18 qt.	0 pt.	1.7 gi.
or, 4 gal.	2 qt.	0 pt.	1.7 gi. Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. $= 14.1$ gi. As 4 gi. = 1 pt., there are as many pints in 14.1 gi. as 4 is contained times in $14.1 = 3.5$ pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. $= 4.7$ pt.; $4.7 + 3.5$ pt. $= 8.2$ pt. As 2 pt. = 1 qt., there are as many quarts in 8.2 pt. as 2 is contained times in $8.2 = 4.1$ qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. $= 14.1$; $14.1 + 4.1 = 18.2$ qt. The answer now is 18.2 qt. 0 pt. .1 gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. $= .2 \times 8 = 1.6$ gi.; $1.6 + .1$ gi. $= 1.7$ gi.). Then, we can reduce 18 qt. to gallons ($18 \div 4 = 4$ gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

3 qt.
× 2 pt.
6 pt.
+ 1 pt.
7 pt.
× 4 gi.
28 gi.
+ 3 gi.
31 gi.

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

4)145.7 gi.
2)36 pt. + 1.7 gi.
18 qt. + 0 pt.

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(110) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

3 lb. 10 oz. 13 pwt. 12 gr
× 12
36 oz.
+ 10
46 oz.
× 20
920 pwt.
+ 13
933 pwt.
× 24
22392 gr.
+ 12
22404 gr.

22,404 gr. × 1.5 = 33,606 gr.

24)33606 gr.
20)1400 pwt. + 6 gr
12)70 oz. + 0 pwt.
5 lb. + 10 oz.

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pwt. as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We can not reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

lb.	oz.	pwt.	gr.	
3	10	13	12	
			1.5	
4.5	15	19.5	18	
or, 4	21	19	30	
or, 5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr., but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the lb. column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(111) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. reserved equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu. for the product of the bushels. 9 times 2 bu. plus the 8 bu. reserved equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

bu.	pk.	qt.
2	3	6
		9
18	27	54
or, 26	1	6

(112) (7 T. 15 cwt. 10.5 lb.) × 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then, multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

7 T. 15 cwt. 10.5 lb.

× 20

140 cwt.

15

155 cwt.

× 100

15500 lb.

10.5

15510.5 lb.

15,510.5 lb. × 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations

100) 26367.85 lb.
20) 263 cwt. + 67.85 lb.
13 T. + 3 cwt.

There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 tons and 3 cwt. remaining. Since we cannot reduce 13 tons any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

(113) 7) 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft.
51 A. 31 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(114) 12) 282 bu. 3 pk. 1 qt. 1 pt.
23 bu. 2 pk. 2 qt. ¼ pt. Ans.

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write

2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(115) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mi. contains 5,280 ft.; hence, 23 mi. contain 5,280 × 23 = 121,440 ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(116) In this case where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either and then divide as in simple numbers.

1 bu. 1 pk. 7 qt.
× 4
4 pk.
+ 1
5 pk.
× 8
40 qt.
+ 7
47 qt.

356 bu. 3 pk. 5 qt.
× 4
1424 pk.
+ 3
1427 pk.
× 8
11416 qt.
+ 5
11421 qt.

47) 11421 (243
94
202
188
141
141

11,421 qt. ÷ 47 qt. = 243 boxes. Ans.

(117) We must first reduce 16 square miles to acres.

In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16 × 640 A. = 10,240 A.

62) 10240 A.
165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in. Ans.

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1{,}600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 times $30\frac{1}{4}$ sq. yd. $= 1{,}512\frac{1}{2}$ sq. yd. 62 is contained in $1{,}512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4{,}968$ sq. in. 62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining.

We write 80 sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

ARITHMETIC.

(SECTION 5.)

(QUESTIONS 118–133.)

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(118) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11{,}664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 1080 \\ \hline 11664 \end{array}$$

(119) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049$. Ans.

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 \\ 9 \\ \hline 729 \\ 9 \\ \hline 6561 \\ 9 \\ \hline 59049 \end{array}$$

(120) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$ Ans.

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Since there are four decimal places in the multiplicand and 4 in the multiplier, we must point off $4+4=8$ decimal places in the product; but as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

```
    .0133
    .0133
    -----
     399
     399
      133
  ---------
  .00017689
    .0133
    -----
     53067
     53067
      17689
  ---------
  .000002352637
```

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8+4=12$ decimal places in the product; but as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(121) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(122) $\sqrt{90}=?$ The root is evidently 9 plus an interminable decimal. Trying 9 for one factor, the other is $90\div 9=10$, and the first approximation is $\frac{9+10}{2}=9.5$. $90\div 9.5=9.473+$, and the second approximation is $\frac{9.5+9.473}{2}=9.486+$, or 9.49 to three figures. Using 9.49 for one factor, the other is $90\div 9.49=9.48366+$, and the third approximation is $\frac{9.49+9.48366}{2}=9.48683$ or $9.4868+$ to five figures. Ans.

This solution may be shortened by using the table and applying the method described in Art. **281** to find the first three significant figures of the root. Referring to the table, the first two significant figures of the root are 9.4; the first difference is $90.25-88.36=1.89$; the second difference is

$90-88.36=1.64$; $1.64\div 1.89=.86+$. Hence, the first three figures are 9.48.

(123) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$\left(3\tfrac{3}{4}\right)^3=\frac{15}{4}\times\frac{15}{4}\times\frac{15}{4}=\frac{15\times 15\times 15}{4\times 4\times 4}=\frac{3.375}{64}=52\tfrac{47}{64}$$

$=52.734375$. Ans.

$$3\tfrac{3}{4}=\frac{3\times 4+3}{4}=\frac{12+3}{4}=\frac{15}{4}.$$

```
    15                64)3375(52 47/64
    15                  320
   ---                  ---
    75                   175
    15                   128
   ---                   ---
   225                    47
    15
   ----
  1125
   225
  ----
  3375
```

```
64)47.000000(.734375
   448
   ---
    220
    192
    ---
     280
     256
     ---
      240
      192
      ---
       480
       448
       ---
        320
        320
        ---
```

Since six ciphers were annexed to the dividend, six decimal places must be pointed off in the quotient.

(124) $\sqrt[3]{92,416} = ?$ Pointing off into periods, the result is 92′416. Since 92 lies between $4^3 = 64$ and $5^3 = 125$, the root is 4+. Trying 4 for one of the two equal factors, the third factor is $92 \div 4^2 = 92 \div 16 = 5.75$. Trying 5 for one of the two equal factors, the third factor is $92 \div 5^2 = 92 \div 25 = 3.68$. Difference between 4 and 5.75 is 1.75, and between 5 and 3.68 is 1.32; hence, use 5, the first approximation being $\frac{2 \times 5 + 3.68}{3} = 4.56$, or 4.6 to two figures.

Using 46 for one of the two equal factors, the third factor is $92416 \div 46^2 = 92416 \div 2116 = 43.67+$, and the second approximation is $\frac{2 \times 46 + 43.67}{3} = 45.22+$, or 45.2 to three figures.

Using 45.2 for one of the two equal factors, the third factor is $92416 \div 45.2^2 = 92416 \div 2043.04 = 45.2345+$, and the third approximation is $\frac{2 \times 45.2 + 45.2345+}{3} = 45.2115+$, or 45.212− to five figures. Ans.

The first three significant figures may also be found by the aid of the table and the method described in Art. **282.**

In order to obtain two figures of the root from the table, we place a decimal point between the first and second significant periods; the result is 92.416. Referring to the table, the first two figures of the root are 4.5; the first difference is $97.336 - 91.125 = 6.211$; the second difference is $92.416 - 91.125 = 1.291$; $1.291 \div 6.211 = .20+$. Therefore, $\sqrt[3]{92.416} = 4.52$ and $\sqrt[3]{92,416} = 45.2$ to three significant figures.

(125) $\sqrt{502,681} = ?$ Pointing off into periods, we have 50′26′81. The first figure of the root is evidently 7, since $7^2 = 49$ and $8^2 = 64$. The two factors then are 7, and $50 \div 7 = 7.14+$. The first approximation is $\frac{7 + 7.14+}{2} = 7.07+$, or 7.1 to two figures. To find the second approximation, we use the first two periods and drop the decimal point in the first

approximation. One factor is then 71 and the other $5026 \div 71 = 70.78+$. The second approximation is therefore $\frac{71 + 70.78}{2} = 70.89$, or 70.9 to three figures. Using 709 for one factor, the other is $502681 \div 709 = 709$. Hence, the number is a perfect power and the root is 709. Ans.

Using the table and placing a decimal point between the first and second periods so that we may obtain two figures of the root from the table, the number becomes $\sqrt{50.2681}$ or to four figures, $\sqrt{50.27}$. Referring to the table, the first two figures of the root are 7.0; the first difference is $50.41 - 49.00 = 1.41$; the second difference is $50.27 - 49.00 = 1.27$; $1.27 \div 1.41 = .9+$. Therefore, $\sqrt{502,681} = 709$ to three figures.

(126) $$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}.$$ Ans.

(127) $4^3 = 4 \times 4 \times 4 = 64.$

$$\sqrt[3]{8} = 2.$$

$$4^3 - \sqrt[3]{8} = 64 - 2 = 62.$$ Ans.

(128) Since $\frac{3}{8} = .375$, $\sqrt[3]{\frac{3}{8}} = \sqrt[3]{.375}$. Moving the decimal point three places to the right, the number becomes 375. Since 375 lies between $7^3 = 343$ and $8^3 = 512$, the root is 7+. Trying 7 for one of the two equal factors, the third factor is $375 \div 7^2 = 375 \div 49 = 7.65+$, and the first approximation is $\frac{2 \times 7 + 7.65}{3} = 7.21+$, or 7.2 to two figures. As the difference between the equal and unequal factors is very slight, it is not necessary to try 8.

Using 7.2 for one of the two equal factors, the third factor is $375 \div 7.2^2 = 375 \div 51.84 = 7.233+$, and the second approximation is $\frac{2 \times 7.2 + 7.233}{3} = 7.211+$, or 7.21 to three figures.

Using 7.21 for one of the two equal factors, the third factor is $375 \div 7.21^2 = 375 \div 51.9841 = 7.21374+$, and the third approximation is $\frac{2 \times 7.21 + 7.21374}{3} = 7.21124+$, or

7.2112+ to five figures. Since the number is entirely decimal, the root is wholly decimal; hence, locating the decimal point, the $\sqrt[3]{.375} = .72112+$. Ans.

By aid of the table, the first three figures are determined as follows: Move the decimal point three places to the right so it will fall between the first period 375 and the cipher period that follows.

Referring to the table, the first two figures of the root are 7.2; the first difference is 389. 17 $-$ 373.248 $=$ 15.769; the second difference is 375.000 $-$ 373.248 $=$ 1.752; 1.752 $\div$ 15.769 $=$ .11+, or .1 to one figure. Hence, the first three figures are 7.21, or the $\sqrt[3]{.375} = .721$. The fourth and fifth figures are then determined as previously indicated.

(129) $\sqrt{.3364} = ?$

Moving the decimal point two places to the right, so that the first period may be integral, the result is 33.64. The first two factors are evidently 5 and $33 \div 5 = 6.6$, and the first approximation is $\frac{5 + 6.6}{2} = 5.8$. $33.64 \div 5.8 = 5.8$. Hence, the given number is a perfect power, and as it is wholly decimal, $\sqrt{.3364} = .58$. Ans.

(130) $\sqrt{3.1416} = ?$

Pointing off, we obtain 3.14'16. The first two significant figures are 3.1. It is evident that the first figure of the root is 1, since $1^2 = 1$ and $2^2 = 4$. Using 1 as one factor, the other is $3.1 \div 1 = 3.1$, and the first approximation is $\frac{1 + 3.1}{2} = 2.05$. Had 2 been used as one factor, the other would have been $3.1 \div 2 = 1.55$, and the first approximati n would have been $\frac{2 + 1.55}{2} = 1.77+$, or 1.8 to two figures. In the first case, the difference between the two factors is $3.1 - 1 = 2.1$; in the second case, the difference is $2 - 1.55 = .45$. As the factors are more nearly equal in the second case than in the first, it is evident that 1.8 is more nearly equal to the correct value of the root than 2.05 is; hence, 1.8 will be used for the first approximation.

For the second approximation, use the first two periods and 1.8 for one factor, the other factor is $3.14 \div 1.8 = 1.74+$; hence, the second approximation $= \frac{1.8 + 1.74}{2} = 1.77$. Using 1.77 for one factor, the other is $3.1416 \div 1.77 = 1.77491+$. The third approximation is $\frac{1.77 + 1.77491}{2} = 1.772455$, or 1.7725— to five figures. Ans.

Using the table to find the first three figures, the first two figures of the root are 1.7; the first difference is $3.24 - 2.89 = .35$; the second difference is $3.14 - 2.89 = .25$; $.25 \div .35 = .71+$. Therefore, $\sqrt{3.1416} = 1.77$ to three significant figures.

(131) Since some number multiplied by itself equals 114.9184, then the number is $\sqrt{114.9184}$. Pointing off into periods and placing the decimal point between the first and second periods, we have 1.14'91'84. Considering the first two figures, it is evident that the first figure of the root is 1. Using 1 as one factor, the other is $1.1 \div 1 = 1.1$, and the first approximation is $\frac{1 + 1.1}{2} = 1.05$, or 1.1 to two figures. Using the first three figures and 1.1 for one factor, the other factor is $1.15 \div 1.1 = 1.045+$, and the second approximation is $\frac{1.1 + 1.045}{2} = 1.072+$, or 1.07 to three figures. Using 1.07 for one factor, the other is $1.149184 \div 1.07 = 1.074003+$, and the third approximation is $\frac{1.07 + 1.074003}{2} = 1.072001+$, or 1.0720 to five figures. Noticing that the square of the last significant figure is $2^2 = 4$, which corresponds to the last figure of the given number, and that the fifth and sixth figures of the third approximation are ciphers, we suspect that the given number is a perfect power. We find such to be the case on squaring 1.072. Since there are two periods in the integral part of the number, there are two figures in the integral part of the root, and $\sqrt{114.9184} = 10.72$. Ans.

Using the table to find the first three figures of the root, the first two figures of the root are 1.0; the first difference

is $1.21 - 1.00 = .21$; the second difference is $1.15 - 1\ 00 = .15$; $.15 \div .21 = .7$. Therefore, the first three figures are 10.7.

(132) $\sqrt{3,486,784} = ?$

Pointing off into periods, we have 3'48'67'84. Placing a decimal point between the first and second periods and using the first two figures, we obtain 3.5. Considering 2 as one factor, the other is $3.5 \div 2 = 1.75$, and the first approximation is $\frac{2+1.75}{2} = 1.87+$, or 1.9 to two figures. For the second approximation, we use the first two periods and 19 for one factor, the other factor being $349 \div 19 = 18.37+$. We used 349 instead of 348 because the fourth figure was 6, and the number correct to three figures is 349. The second approximation is $\frac{19+18.37}{2} = 18.68+$, or 18.7 to three figures. $34868 \div 187 = 186.459+$; $\frac{187 + 186.459}{2} = 186.729+$, or 1867.3— to five figures. Ans.

In order to obtain three figures from the table, we place the decimal point between the first and second periods, and use the first two periods only; that is, we find the value of $\sqrt{3.49}$. Referring to the table, the first two figures of the root are 1.8; the first difference is $3.61 - 3.24 = .37$; the second difference is $3.49 - 3.24 = .25$; $.25 \div .37 = .67+$, or .7 to one figure. Therefore, $\sqrt{3,486,784} = 187$ to three significant figures.

The fourth and fifth figures may be found as previously indicated.

(133) $\sqrt{.00041209} = ?$

Pointing off into periods, the result is .00'04'12'09. Placing the decimal point between the first and second periods of the significant part of the number, we obtain 4.1 for the first two figures. The first factor is evidently 2 and the second factor $4.1 \div 2 = 2.05$. The first approximation is $\frac{2+2.05}{2} = 2.02+$, or 2.0 to two figures. Using 20 for one factor, the other is

$412 \div 20 = 20.6$, and the second approximation is $\frac{20 + 20.6}{2} = 20.3$.

Using 203 for one factor, the other is $41209 \div 203 = 203$. Hence the number is a perfect power and the significant figures of the root are 203. There being one full cipher period following the decimal point, the root is .0203. Ans.

Using the table, we find that the first two figures of the root are 2.0; the first difference is $4.41 - 4.00 = .41$; the second difference is $4.12 - 4.00 = .12$; $.12 \div .41 = .3$, nearly. Therefore, $\sqrt{.00041209} = .0203$ to three significant figures.

ARITHMETIC.

(SECTION 6.)

(QUESTIONS 143–167.)

---

(143) $11.7 : 13 :: 20 : x.$ The product of the means equals the product of the extremes.

$$11.7\,x = 13 \times 20$$
$$11.7\,x = 260$$
$$x = \frac{260}{11.7}$$

$11.7\overline{)260.000}\,(22.22+$ Ans.

234

260

234

260

234

260

234

26

(144) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$$27 : 18 :: 3 : x$$
$$27\,x = 18 \times 3$$
$$27\,x = 54$$
$$x = \frac{54}{27} = 2. \quad \text{Ans.}$$

(*b*) $12^2 : 100^2 :: 4 : x.$

$$144 : 10,000 :: 4 : x$$
$$144\,x = 10,000 \times 4$$
$$144\,x = 40,000$$

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$$x = \frac{40{,}000}{144}$$

144) 40000.0 (277.7 + **Ans**

288

1120

1008

1120

1008

1120

1008

112

(145) (*a*) $\frac{4}{x} = \frac{7}{21}$, is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \textbf{Ans.}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \textbf{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x.$

$15x = 45 \times 60$

$15x = 2{,}700$

$x = \frac{2{,}700}{15} = 180.$ Ans.

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600.$

$150x = 10 \times 600$

$150x = 6{,}000$

$x = \frac{6{,}000}{150} = 40.$ Ans.

(146) $x : 5 :: 27 : 12.5.$

5

12.5) 135.0 ($10\frac{4}{5}$ Ans.

125

100

125 $= \frac{4}{5}$

(147) $45 : 60 :: x : 24$

$60x = 45 \times 24$

$60x = 1{,}080$

$x = \frac{1{,}080}{60} = 18.$ Ans.

(148) $x : 35 :: 4 : 7.$

$7x = 35 \times 4$

$7x = 140$

$x = \frac{140}{7} = 20.$ Ans.

(149) $9 : x :: 6 : 24.$

$6x = 9 \times 24$

$6x = 216$

$x = \frac{216}{6} = 36.$ Ans

(150)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$10x = 297.$

$x = \frac{297}{10} = 29.7.$ Ans.

1

1

2

1

30

1

31

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

1 1'331 (11

2 1

300 331

31 331

331

(151) $64 : 81 = 21^2 : x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} = \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$8 : 9 = 21 : x$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(152) $7 + 8 : 7 = 30 : x$ is equivalent to

$15 : 7 = 30 : x.$

$15x = 7 \times 30$

$15x = 210$

$x = \frac{210}{15} = 14.$ **Ans.**

(153) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, $29 : 31 = 2{,}480 : x$. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since, when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29 : 31 = x : 2{,}480,$$
$$\text{or,} \quad 31x = 71{,}920;$$
$$\text{whence,} \quad x = 71{,}920 \div 31 = 2{,}320 \text{ steps.} \quad \textbf{Ans.}$$

(154) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96 : 900 = 12 : x,$$
$$\text{or,} \quad 96x = 10{,}800;$$
$$\text{whence,} \quad x = 10{,}800 \div 96 = 112.5 \text{ mi.} \quad \textbf{Ans.}$$

(155) This is also a direct proportion; hence,

$$27.63 : 29.4 = .76 : x,$$
$$\text{or,} \quad 27.63x = 29.4 \times .76 = 22.344;$$
$$\text{whence,} \quad x = 22.344 \div 27.63 = .808 + \text{lb.} \quad \textbf{Ans.}$$

(156) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt. Hence,

$$23 : 46 = 5 : x,$$
$$\text{or,} \quad 23x = 46 \times 5 = 230;$$
$$\text{whence,} \quad x = 230 \div 23 = 10 \text{ days.} \quad \text{Ans.}$$

(157) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2 : 12^2 = 24 : x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2 : 12^2 = x : 24.$$

Dividing both terms of the first couplet (see Art. **310**) by 6

$$1^2 : 2^2 = x : 24; \text{ or } 1 : 4 = x : 24;$$
$$\text{whence, } 4x = 24, \text{ or } x = 6 \text{ degrees.} \quad \text{Ans.}$$

(158) Taking the dimensions as the causes,

$$\begin{array}{c|c} \cancel{12} & 15 \\ \cancel{4} & 5 \\ 2 & \cancel{2} \\ \cancel{3} & 6 \end{array} = \cancel{12} \Big| \; x, \text{ whence, } 2x = 75, \text{ or, } x = \$37.50. \quad \text{Ans.}$$

(159) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min.

$$\text{Hence,} \quad 120 : 868 = 100 : x,$$
$$\text{or,} \quad 120x = 86{,}800;$$
$$\text{whence,} \quad x = 723\tfrac{1}{3} \text{ gal.} \quad \text{Ans.}$$

(160) Taking the dimensions as the causes,

$$\begin{array}{c|cc} \cancel{14} & \cancel{2} & \\ \cancel{28} & \cancel{20} & \\ 2 & & = \cancel{798} \\ \cancel{12} & 17 & 57 \\ \cancel{10} & \cancel{6} & \end{array} \Big| \; x, \text{ whence, } 2x = 17 \times 57 = 969,$$
$$\text{or, } x = 484\tfrac{1}{2} \text{ bbl.} \quad \text{Ans}$$

(161) 8 hr. 40 min. = 520 min. Hence,

$$444 : 1{,}060 = 520 : x,$$

$$\text{or, } x = \frac{1{,}060 \times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{137{,}800}{111} = 1{,}241.44 + \text{ min.} = 20 \text{ hr.}$$
$$41.44 + \text{ min.} \quad \text{Ans.}$$

(162) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2} : 60 = 6{,}160 : x,$$
$$\text{or, } x = \frac{60 \times 6{,}160}{5.5} = 67{,}200 \text{ ft.} \quad \text{Ans.}$$

(163) Writing the statement as a direct proportion, $8 : 10 = 5 : x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8 : 10 = x : 5,$$

$$\textbf{or,} \quad x = \frac{\overset{4}{\cancel{8}} \times 5}{\underset{2}{\cancel{10}}} = 4 \text{ men.} \quad \textbf{Ans.}$$

(164) Taking the times as the causes,

$$\begin{array}{|cc|c|} \not{2}\not{0} & \not{2}\not{5} & 14 \\ & \not{5} & \not{7}\not{0} \\ & = \not{5}\not{4}\not{0} & \not{6}\not{3}\not{0} \\ \not{1}\not{0} & x \quad \not{2}\not{7} & \\ 2 & 3 & \end{array}$$

; whence, $3x = 2 \times 14 = 28$, or $x = 9\frac{1}{3}$ hr. Ans.

(165) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **349,** 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{|cc|c|} 14^2 & 30^2 & \\ 500 & 660 = 112 & x, \text{ or} \\ 48 & 42 & \end{array} \quad \begin{array}{|cc|} & 9 \\ \not{1}\not{4} & \not{9}\not{0}\not{0} \\ \not{1}\not{9}\not{6} & 22 \\ \not{5} & \not{1}\not{1}\not{0} \\ \not{5}\not{0}\not{0} & \not{6}\not{6}\not{0} = \not{1}\not{1}\not{2} \\ \not{6} & 3 \quad 8 \\ \not{4}\not{8} & \not{4}\not{2} \end{array} \quad x;$$

whence, $x = 9 \times 22 \times 3 = 594$ horsepower. Ans.

(166) First find the volume of the cylinder in cubic inches, as in the example, Art. **345.** The volume, multiplied by the weight of one cubic inch (.261 lb.), will evidently be the weight of the cylinder. Hence,

$$\begin{array}{|cc|c|} 10^2 & 12^2 & \\ 20 & 60 & = 1{,}570.8 \end{array} \quad x, \text{ or} \quad \begin{array}{|cc|} 100 & 144 \\ & 3 \quad = 1.570.8 \\ \not{2}\not{0} & \not{6}\not{0} \end{array} \quad x;$$

whence, $x = \dfrac{144 \times 3 \times 1{,}570.8}{100} = 6{,}785.856$ cu. in. Therefore, weight of cylinder $= 6{,}785.856 \times .261 = 1{,}771.11$ lb. Ans.

(167) Referring to the example in Art. **348,**

$$\begin{array}{|cc|c|} 15 & 40 & \\ & & \\ 20^2 & 18^2 = 187 & x, \text{ or} \\ 10 & 12 & \end{array} \quad \begin{array}{|cc|} & \not{4} \\ 5 & \not{4}\not{0} \\ \not{1}\not{5} & \\ 100 & \\ \not{4}\not{0}\not{0} & 324 = 187 \\ \not{1}\not{0} & 4 \\ & \not{1}\not{2} \end{array} \quad x;$$

whence, $x = \dfrac{324 \times 4 \times 187}{500} = 484.7$ lb. Ans.

NOTICE.

The present set of answers on the subject of Arithmetic are less in number than were contained in the former edition. As a consequence, there is a slight break in the page numbers between the last page of the answers on Arithmetic and the paper following.

MENSURATION AND USE OF LETTERS IN FORMULAS.

(QUESTIONS 142–213.)

(142) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(143) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(144) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246 \times B \times h}{\frac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\frac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\frac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269 +. \quad \text{Ans.}$$

(145) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05 +. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

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(146) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{Bx}{.00018\,h\,(A^2 - x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41} + = 6.35 +. \quad \text{Ans.}$$

(147) Substituting for h, D, and A their values,

$$f = \frac{10(h-D)^2}{\sqrt[3]{D+A}} = \frac{10\,(200-120)^2}{\sqrt[3]{120+5}} = \frac{10\times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5} = 12{,}800$$

Ans.

(148) Substituting for B, A, and D their values,

$$g = \frac{(B-A)^2 - \sqrt[3]{D+A}}{A^3 - (1+D)} = \frac{(10-5)^2 - \sqrt[3]{120+5}}{5^3 - (1+120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25 - 5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(149) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071+. \quad \text{Ans.}$$

(150) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(hx)^2}{D^2}\right]}{h + \frac{x}{D}(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22{,}250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

(151) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles = 152° 3′, the other angle = 180° − 152° 3′, or

$$180° = 179°\ 60'$$
$$\text{subtracting,}\quad 152°\ 3'$$
$$27°\ 57'. \quad \text{Ans.}$$

(152) There are 60 seconds in one minute and 60 minutes in one degree; therefore, 140° = 140 × 60 = 8,400 minutes; 8,400′ + 17′ = 8,417′; 8,417′ = 8,417 × 60 = 505,020 seconds, and 505,020″ + 10″ = 505,030″. Ans.

(153) See Arts. **359** and **360.**

(154) (*a*) 240 ÷ 60 = 4, the number of degrees. Ans.

(*b*) 240 × 60 = 14,400, the number of seconds. Ans.

(155) See Arts. **355-357.**

(156) See Art. **369.** If the rectangle has the same base and altitude, it would have the same area.

(157) No, since the sum of the three shorter sides is not greater than the fourth side.

(158) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area = 30 × $11\frac{1}{2}$ = 345 sq. in. Ans.

(159) Since there are 144 sq. in. in 1 sq. ft., the area of the zinc in the last example $= \frac{345}{144} = 2.396$ sq. ft. The weight per square foot $= \frac{5.25}{2.396} = 2.19+$ lb. Ans.

(160) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide, and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required = 36 × $1\frac{1}{2}$ = 54. Ans.

(161) By the rule for the area of a trapezoid, area of the land $= \frac{9+6}{2} \times 16 = 120$ square rods. Since there are 160 square rods in one acre, 120 square rods $= \frac{120}{160} = \frac{3}{4}$ of an acre. Ans.

(162) The total area of the floor of the station = 55 × 58 ft. = 3,190 sq. ft. − 25 × 26 ft. = 650 sq. ft., the area represented by the lower right-hand corner of the

figure. Hence, total area of floor = 3,190 − 650 = 2,540 sq. ft.

From this we have to deduct the following areas:

2 boilers	= 2 × 8 × 19	= 304.	sq. ft.
Feed pump	= $2\frac{1}{2} \times 5$	= 12.5	sq. ft.
2 engines	= $2 \times 4\frac{1}{2} \times 10$	= 90	sq. ft.
2 dynamos	= $2 \times 5\frac{1}{2} \times 6\frac{1}{2}$	= 71.5	sq. ft.
Switchboard	= $\frac{10 \times 3.5}{12}$	= 2.92	sq. ft.
		480.92	sq. ft.

The unoccupied floor space, therefore, =

2,540 − 480.92 = 2,059.08 sq. ft. Ans.

(163) The length of the walk, neglecting the corners, that is, the distance $a\,b + b\,c + c\,d + d\,a$, Fig. 1, = 2 × 528 + 2 × 352 = 1,760 ft. The width = 10 ft.; area = 1,760 × 10 = 17,600 sq. ft. The area of the four corners = 4×10^2 = 400 sq. ft. Total area of walk = 17,600 + 400 = 18,000 sq. ft. Hence the area in sq. yds. = $\frac{18,000}{9}$ = 2,000. Ans.

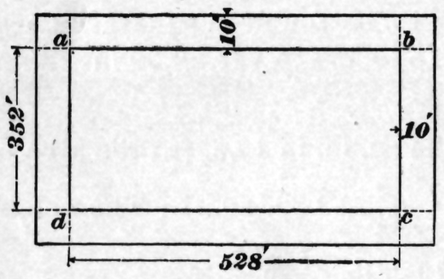

FIG. 1.

(164) The area of the sides of the room = 2 × 20 × 90 = 3,600 sq. ft.; area of ends = 2 × 20 × 50 = 2,000 sq. ft. Total area of walls = 3,600 + 2,000 = 5,600 sq. ft.

From this is to be deducted the following areas:

4 doors = 4 × $5\frac{1}{2}$ × 10 = 220 sq. ft.

14 windows = 14 × 5 × 11 = 770 sq. ft.

Length of baseboard, deducting the width of the 4 doors = 2 × 90 + 2 × 50 − 4 × $5\frac{1}{2}$ = 180 + 100 − 22 = 258 feet; width of baseboard = 9 inches, or $\frac{3}{4}$ of a foot; area of baseboard = $258 \times \frac{3}{4}$ = 193.5 sq. ft. Hence, we have to deduct 220 + 770 + 193.5 = 1,183.5 sq. ft.

Number of sq. ft. of plastering = 5,600 − 1,183.5 = 4,416.5 sq. ft.; number of sq. yd. of plastering = 4,416.5 ÷ 9 = 490.72. Ans.

(165) A triangle with three equal angles has three equal sides, and is, therefore, an equilateral triangle.

(166) A triangle with two equal angles has two equal sides, and is, therefore, an isosceles triangle.

(167) No, since the sum of the two shorter sides is not greater than the third side.

(168) (*a*) Draw a line $B\,D$ from the vertex perpendicular to the base, Fig. 2. It will divide the base into two equal parts, as shown. In the right-angled triangle $A\,B\,D$, the hypotenuse $A\,B = 6$, and side $A\,D = 3$; hence, by rule **50**, Art. **385**, $B\,D$, the altitude = $\sqrt{6^2 - 3^2} = \sqrt{27} = 5.196$ ft. Ans.

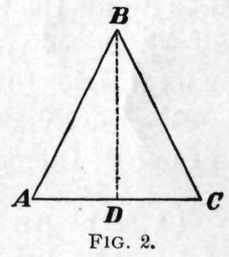

FIG. 2.

(*b*) Area = $\frac{6 \times 5.196}{2}$ = 15.588 sq. ft. Ans.

(169) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles = 23° + 32° 32′ = 55° 32′, and the third angle = 180° − 55° 32′, or

180° = 179° 60′

subtracting, 55° 32′

124° 28′. Ans.

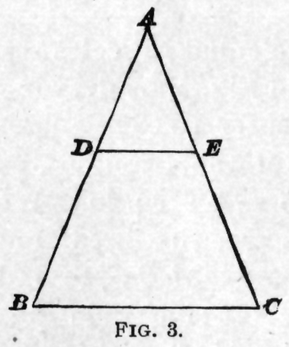

FIG. 3.

(170) In Fig. 3 we have the proportion $A\,D : D\,E :: A\,B : B\,C$, in which $A\,D = 10$ in., $A\,B = 24$ in., and $B\,C = 13\frac{1}{2}$ in., to find $D\,E$.

Substituting the given values,

10 : $D\,E$ :: 24 : $13\frac{1}{2}$, or

$D\,E = \frac{10 \times 13.5}{24}$ = 5.625 in. Ans.

(171) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right-angled

triangle, whose two sides are 39 and 52 feet. By rule **49,** Art. **385,** the length of the diagonal $= \sqrt{52^2 + 39^2} = 65$ ft. Ans.

(172) Using rule **50,** Art. **385,** the required distance $= \sqrt{24^2 - 8^2} = \sqrt{576 - 64} = 22.627$ ft. $= 22$ ft. $7\frac{1}{2}$ in. $+$. Ans.

(173) See Art. **386.**

(174) (*a*) Using rule **52,** Art. **386,** length of base $= \frac{2 \times 200}{20} = 20$ in. Ans.

(*b*) A perpendicular let fall from the vertex to the base will divide the triangle into two equal right-angled triangles, of which the perpendicular side is 20 inches and the horizontal side 10 inches. Hence, according to rule **49,** Art. **385,** the hypotenuse $= \sqrt{20^2 + 10^2} = 22.36$ in. $=$ the length of one of the equal sides of the given triangle. Ans.

(175) An equilateral heptagon has seven equal sides; hence, the sum of all the sides, or the perimeter, $= 7 \times 3 = 21$ in. Ans.

(176) A regular decagon has 10 equal sides; hence, the length of one of the sides $= 40 \div 10 = 4$ inches. Ans.

(177) A dodecagon has 12 sides, and by rule **53,** Art. **389,** we have interior angle $\frac{180 \times (12 - 2)}{12} = 150°$. Ans.

(178) Divide the pentagon into five equal isosceles triangles, as shown in Fig. 4, by drawing a line from the center to each angle. The area of one of the triangles, as $A\ B\ C$, $= 43 \div 5 = 8.6$ sq. in. We have given, therefore, the area and base $B\ C$ of the triangle $A\ B\ C$, to find its altitude $A\ D$. Using rule **52,** Art. **386,** $A\ D = \frac{8.6 \times 2}{5} = 3.44$ in., the perpendicular distance from the center to one side. Ans.

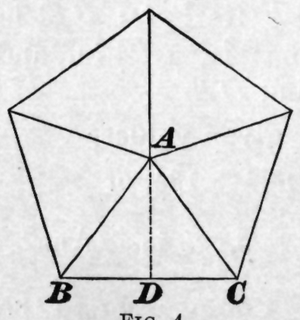

FIG. 4.

(179) See example, Art. **389.** The process is simply to find one of the angles of the polygon, and then to divide it by 2. By rule **53,** Art. **389,** one of the interior angles $= \frac{180 \times (8 - 2)}{8} = 135°$. This divided by $2 = 67\frac{1}{2}°$. Ans.

(180) Divide the figure into two triangles and one trapezoid, as shown in Fig. 5. The dimensions of the different parts can then be found by measurement. In obtaining the areas, it will be easier to use decimals than common fractions.

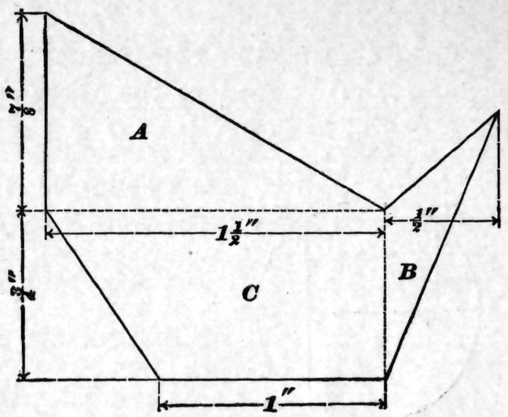

FIG. 5.

Area of triangle $A = \frac{1\frac{1}{2} \times \frac{7}{8}}{2} = \frac{1.5 \times .875}{2} = .656$ sq. in.

Area of triangle $B = \frac{\frac{3}{4} \times \frac{1}{2}}{2} = \frac{.75 \times .5}{2} = .188$ sq. in.

Area of trapezoid $C = \frac{1\frac{1}{2} + 1}{2} \times \frac{3}{4} = \frac{1.5 + 1}{2} \times .75 = 1.25 \times .75 = .938$ sq. in.

Hence, the area of the whole figure $= .656 + .188 + .938 = 1.78 +$ sq. in. Ans.

(181) An angle inscribed in a circle is measured by one-half the intercepted arc. In this case, the angle intercepts one-fourth the circumference, and is measured by one-eighth the circumference, or by $360° \times \frac{1}{8} = 45°$. Hence, there are 45° in the angle. Ans.

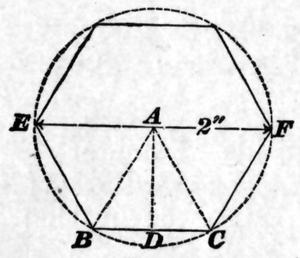

FIG. 6.

(182) Since this is a regular hexagon, it may be inscribed in a circle (Fig. 6), and the radius of the inscribing circle will be equal to one side of the hexagon. Since the diameter $E\ F = 2$ inches, the radii

$A\,B$ and $A\,C$, and the side $B\,C$ each = 1 inch, and the triangle $A\,B\,C$ is equilateral. Draw the line $A\,D$ perpendicular to the side $B\,C$; it will bisect $B\,C$. Then, in the right-angled triangle $A\,D\,B$, $A\,B = 1$, and $B\,D = \frac{1}{2}''$, to find $A\,D$. According to rule **50,** Art. **385,** $A\,D = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= A\,D \times 2 = .866 \times 2 = 1.732''$. Ans.

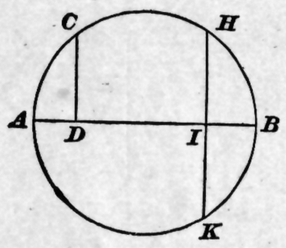

FIG. 7.

(183) In Fig. 7, we have the proportion $B\,I : H\,I :: H\,I : I\,A$, in which $B\,I = 6$, and $H\,I = \frac{1}{2}$ of $H\,K = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : I\,A$, or $I\,A = \frac{81}{6} = 13.5$ in. Hence, the diameter $A\,B = I\,A + B\,I = 13.5 + 6 = 19.5$ in. Ans.

(184) If the diameter $A\,B = 32\frac{1}{2}$ ft. and $I\,B = 8$ ft., $A\,I = 32\frac{1}{2} - 8 = 24\frac{1}{2}$ ft. Then, from the proportion of the last example, $8 : H\,I :: H\,I : 24.5$, whence $H\,I = \sqrt{8 \times 24.5} = \sqrt{196} = 14$ ft., and $H\,K = 2 \times 14 = 28$ ft. Ans. (See Art. **400.**)

(185) Diameter = circumference ÷ 3.1416. By rule **56,** Art. **403,** diameter of tree = 7.854 ÷ 3.1416 = $2\frac{1}{2}$ ft. Ans.

(186) One mile = 5,280 feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **55,** Art. **402.**) Number of revolutions in going one mile = 5,280 ÷ 18.8496 = 280.112. Ans.

(187) Using rule **58,** Art. **405,** area = diameter squared × .7854. $6.06^2 = 36.7236$: $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(188) By rule **59,** Art. **406,** inside diameter = $\sqrt{\frac{113.0976}{.7854}} = 12''$. Since the pipe is $\frac{3}{4}''$ thick, the outside diameter must be $\frac{3}{4}'' \times 2 = 1\frac{1}{2}''$ more, or $12 + 1\frac{1}{2} = 13\frac{1}{2}''$. Ans.

(189) Since the radius of the circle = 6 in., its diameter = 12 in., and its circumference = $12 \times 3.1416 = 37.6992$ in. There are 360° in the circumference, and the length of an arc of 12° $= 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(190) The area of a circle 22 inches in diameter = $22^2 \times .7854 = 380.1336$ sq. in. (See rule **58,** Art. **405.**) Area of a circle 21 inches in diameter = $21^2 \times .7854 = 346.3614$ sq. in. Hence, the area of a flat ring whose outside diameter = 22 in. and inside diameter = 21 in. is 380.1336 − 346.3614 = 33.7722 sq. in. Ans.

(191) In the formula of rule **61,** Art. **408,** $\frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}$, h = the height of the segment = 5 in., and D = the diameter of the circle = 56 in. Hence, the area of the segment $= \frac{4 \times 5^2}{3}\sqrt{\frac{56}{5} - .608} = \frac{100}{3}\sqrt{10.592} = 33\frac{1}{3} \times 3.255 =$ 108.5 sq. in. Ans.

(192) In Fig. 8, let $A\,C\,B\,D$ represent a section of the largest square bar that can be planed from the round bar. $A\,B$ and $C\,D$ each = 2 in., and in the right-angled triangle $A\,O\,C$, the sides $A\,O$ and $C\,O$ each = 1 in. The hypotenuse $A\,C = \sqrt{1^2 + 1^2} = \sqrt{2} = 1.4142$ in. Ans. Hence, the largest square bar that can be planed from the round bar is $1.4142''$ square.

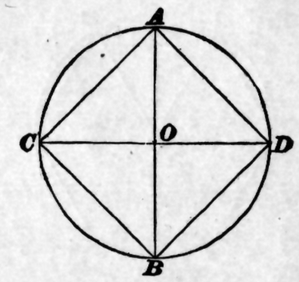

FIG. 8.

(193) The area of a circle 15 in. in diameter = $15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle, whose angle is $12\frac{1}{2}°$, $= 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2{,}208.937}{360} =$ 6.1359 sq. in. Ans. (See rule **60,** Art. **407.**)

(194) (*a*) The side of a square whose area = 103.8691 sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **59,** Art. **406,** the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square $= 10.1916 \times 4 = 40.7664$ in.; circumference of the circle $= 11.5 \times 3.1416 = 36.1284$ in.; difference $= 40.7664 - 36.1284 = 4.638$ in. Ans.

(195) With 5 in. allowed for lap, the length of the plate which forms the shell is $46 - 5 = 41$ in. This is the circumference of the shell. By rule **56,** Art. **403,** the diameter corresponding to this circumference is $\frac{41}{3.1416} =$ 13.05 in. Ans.

(196) The convex area is the area of the outside surface, not including the area of the ends. The circumference of the base $= 26 \times 3.1416 = 81.6816$ in. This reduced to feet, since the area is to be in feet, $= 81.6816 \div 12 = 6.8068$. Using rule **62,** Art. **416,** convex area $= 6.8068 \times 10\frac{1}{2} =$ 71.4714 sq. ft. Ans.

(197) The perimeter of the base $= 4 \times 6 = 24$ in. $= 2$ ft. Convex area $= 2 \times 12 = 24$ sq. ft. The area of the bases is found as follows: In Fig. 9, $AB = 4$ in. and $AC = 2$ in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **50,** Art. **385,** $OC = \sqrt{4^2 - 2^2} = \sqrt{12} = 3.4641$ in.; area of triangle $AOB = \frac{4 \times 3.4641}{2} = 6.9282$ sq. in.; area of base $= 6.9282 \times 6 = 41.5692$, and the area of both bases $= 41.5692 \times 2 = 83.1384$ sq. in. This reduced to square feet $= \frac{83.1384}{144} = .5774$. Hence, the area of the entire surface of the column is $24 + .5774 = 24.5774$ sq. ft. Ans.

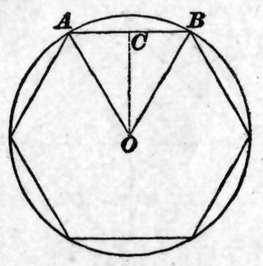

FIG. 9.

(198) The cubical contents in cubic inches = area of base in square inches × altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.; altitude $= 12 \times 12 = 144$ in. Hence, the cubical contents $= 41.5692 \times 144 = 5{,}985.9648$ cu. in. Ans.

(199) This example is solved by combining the rules for the circular ring (see example, Art. **406**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854 = 12.5664$ = area of a circle 4 inches in diameter; $3.73^2 \times .7854 = 10.9272$ = area of a circle 3.73 inches in diameter; difference $= 12.5664 - 10.9272 = 1.6392$ = area of one end of the tube. The cubical contents $= 1.6392 \times 12 = 19.6704$ cu. in.; the weight $= 19.6704 \times .28 = 5.5$, or $5\frac{1}{2}$ lb. Ans.

(200) This example is done exactly like the one in Art. **417,** and the solution is given here without explanation.

(*a*) In the formula of rule **61,** Art. **408,** $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case $= 18$, and $D = 60$.

Substituting, area $=$

$$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725} =$$

$432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $= 60^2 \times .7854 = 2{,}827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes $= 3.5^2 \times .7854 \times 64 = 615.75$ sq. in.
Area of segment $=$ 712.8 sq. in.
1,328.55 sq. in.

Area of water space $= 2{,}827.44 - 1{,}328.55 = 1{,}498.89$ sq. in.

Contents of water space $= 1{,}498.89 \times 16 \times 12 = 287{,}786.88$ cu. in., and $287{,}786.88 \div 231 = 1{,}245.83$, number of gallons, or say 1,246 gal. Ans.

(201) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet (rule **58,** Art. **405**).

Length of stroke plus the clearance = 1.14 × 2 ft. (24 in. = 2 ft.) = 2.28 ft.

1.9689 × 2.28 = 4.489 cubic feet, or the volume of steam in the small cylinder (rule **63**, Art. **417**).

(*b*) Area of piston = $31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance = 1.08 × 2 = 2.16 ft.

5.2414 × 2.16 = 11.321 cubic feet, or the volume of steam in the large cylinder. Ans.

(*c*) Ratio = $\frac{11.321}{4.489}$ or 2.522 : 1. Ans.

(202) Let the equilateral triangle *A B C*, Fig. 10, represent the base of the pyramid. By rule **50**, Art. **385**, the altitude *A D* of the triangle = $\sqrt{10^2 - 5^2} = \sqrt{75} = 8.6602$ in., and according to rule **51**, Art. **386**, the area of the triangle = $\frac{10 \times 8.6602}{2} = 43.301$ sq. in.

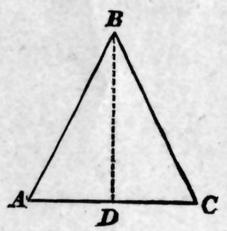

FIG. 10.

Using rule **65**, Art. **423**, volume of pyramid = area of base × $\frac{1}{3}$ altitude = $\frac{43.301 \times 10}{3} = 144.336$ cu. in. Ans.

(203) In Fig. 11, let *O H* be the altitude and *O E* the slant height of the pyramid. Connect points *H* and *E*, forming the right-angled triangle *O H E*, in which we have to find *O H*. Since it is a right pyramid, point *H* will fall at the center of the base *A B C D*, and, hence, the line $HE = \frac{1}{2} AB$, or 8 in.; *O E* = 25 in.; by rule **50**, Art. **385**, $OH = \sqrt{25^2 - 8^2} = \sqrt{561} = 23.6854$ in. Ans.

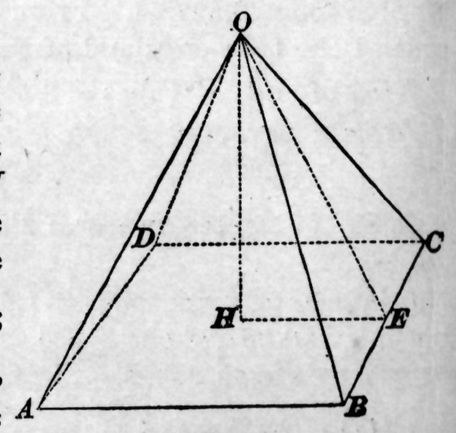

FIG. 11.

(204) The area of the convex surface = circumference of base × $\frac{1}{2}$ slant height = 18.8496 × $\frac{10}{2}$ = 94.248 sq. in. (See rule **64**, Art. **422.**) The area of the entire surface = 94.248 sq. in. + the area of the base. The diameter of the base = $\frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base = $6^2 \times .7854 = 28.2744$ (rules **56** and **58**, Arts. **403** and **405**); therefore, the area of the entire surface = 94.248 + 28.2744 = 122.5224 sq. in. Ans.

(205) Using rule **65**, Art. **423**, volume = area of base × $\frac{1}{3}$ altitude = 28.2744 × $\frac{9}{3}$ = 84.8232 cu. in. Ans.

(206) The vat has the form of an inverted frustum of a pyramid. Area of larger base = $15^2 = 225$ sq. ft.; area of smaller base = $12^2 = 144$ sq. ft. Hence, by rule **67**, Art. **427**, the contents of the vat in cubic feet = $(225 + 144 + \sqrt{225 \times 144})\frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3} =$ 2,013 cu. ft. This should be reduced to cubic inches by multiplying by 1,728, the number of cubic inches in a cubic foot. 2,013 × 1,728 = 3,478,464 cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold = $\frac{3,478,464}{231} = 15,058.29$. Ans.

(207) The pail is in the form of a frustum of a cone. Area of larger base = $12^2 \times .7854 = 113.0976$ sq. in. Area of smaller base = 63.6174 sq. in. Hence, the contents in cubic inches =

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3} =$$

$$(176.715 + \sqrt{7,194.9753})\frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3} =$$

$$261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the

last example to be 3,478,464. Hence, the number of pails of water required to fill the vat = 3,478,464 ÷ 958.9734 = 3,627.28. Ans.

(208) By rule **66,** Art. **426,** the area of the convex surface = half the sum of the perimeters of the upper and lower bases × the slant height, or $\frac{48+36}{2} \times 32 = 42 \times 32 =$ 1,344 sq. in.

(209) See note following the question.

Outside diameter of upper base $= \frac{170.5}{3.1416}$ = about 54.27 in.; inside diameter = 54.27 − 2.5 = 51.77 in.; area of upper base = $51.77^2 \times .7854 =$ 2,105 sq. in., nearly.

Outside diameter of lower base $= \frac{190}{3.1416} = 60.48$ in., nearly; inside diameter = 60.48 − 2.5 = 57.98 in.; area of lower base = $57.98^2 \times .7854 =$ 2,640 sq. in., nearly.

Apply rule **67,** Art. **427.** Contents of the tank in cubic inches = $(2{,}105 + 2{,}640 + \sqrt{2{,}105 \times 2{,}640}) \times \frac{7 \times 12}{3} = (4{,}745$ + 2,357) × 28 = 7,102 × 28 = 198,856 cu. in. Hence, the number of gallons = 198,856 ÷ 231 = 860.8. Therefore, in round numbers, the tank will hold 861 gallons.

(210) (*a*) By rule **68,** Art. **429,** area of the surface = $22.5^2 \times 3.1416 = 506.25 \times 3.1416 =$ 1,590.435 sq. in. Ans.

(*b*) Using rule **69,** Art. **430,** the cubical contents = the cube of the diameter × .5236 = 11,390.625 × .5236 = 5,964.1313 cu. in. Ans.

(211) Having given the area of the surface, to find the volume we must first obtain the diameter. The process is just the reverse of finding the surface when the diameter is given. Hence, the diameter = $\sqrt{201.0624 \div 3.1416} = \sqrt{64} =$ 8 in. By rule **69,** Art. **430,** volume = $8^3 \times .5236 =$ 268.0832 **cu. in. Ans.**

(212) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. 12):

Radius of center circle equals $\frac{8+6.5}{2}$, or $7\frac{1}{4}$ inches.

Length of center line = 2 × 3.1416 × $7\frac{1}{4}$ = 45.5532 inches.

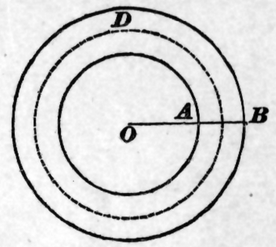

FIG. 12.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **70,** Art. **431,** the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 =$ 214.665 square inches. Ans.

Diameter of cross-section of ring = $1\frac{1}{2}$ inches.

Area of cross-section of ring = $(1\frac{1}{2})^2 \times .7854 =$ 1.76715 sq. in. Ans.

By rule **71,** Art. **432,** volume of ring = 1.76715 × 45.553 = 80.499 cu. in. Ans.

(*b*) Weight of ring = 80.499 × .261 = 21 lb. Ans.

(213) The volume of the ring equals the product of its cross-sectional area and its length on line D; therefore,

cross-sectional area $= \frac{144.349}{20.42} =$ 7.069 sq. in.

The diameter $AB = \sqrt{\frac{7.069}{.7854}} =$ 3 in.

Area of surface = circumference of circle, of which AB is diameter, × length = 3.1416 × 3 × 20.42 = 192.45 square inches. Ans.

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